Outline Solutions to Tutorial Sheet 9

1. (i) Each component has reliability function $R(t) = e^{-0.1t}$ so the system has reliability function

$$R_s(t) = e^{-0.1t} \times e^{-0.1t} \times e^{-0.1t} \times e^{-0.1t} \times e^{-0.1t} = e^{-0.5t}.$$

Then $R_s(5) = e^{-2.5} = 0.082$.

(ii) The reliability function for k of these components in series is $R_s(t) = e^{-k(0.1)t}$ so $R_s(5) = e^{-0.5k}$. Then, since $e^{-0.5 \times 2} = 0.368$ and $e^{-0.5 \times 3} = 0.223$, only 2 can be connected in series.

(iii) Expected lifetime $= \int_0^\infty R_s(t)dt = \int_0^\infty e^{-0.5t}dt = \left[\frac{e^{-0.5t}}{-0.5}\right]_0^\infty = (0) - \left(\frac{1}{-0.5}\right) = 2$

i.e. Expected lifetime is 2000 hours.

2. (i) $R_s(t) = 1 - (1 - e^{-1t})(1 - e^{-3t})(1 - e^{-4t})(1 - e^{-6t})$.

Then $R_s(1) = 1 - (1 - e^{-1})(1 - e^{-3})(1 - e^{-4})(1 - e^{-6})$

$= 0.412$

Thus, the probability the system fails within 1 year is 1 – 0.412 = 0.588.

(ii) Each component has $R(t) = e^{-2t}$ so k or these in parallel has

$$\begin{aligned} R_{s'}(t) &= 1 - (1 - e^{-2t})(1 - e^{-2t}) \cdots (1 - e^{-2t}) \\ &= 1 - (1 - e^{-2t})^k \end{aligned}$$

and, at one year,

$$\begin{aligned} R_{s'}(1) &= 1 - (1 - e^{-2})^k \\ &= 1 - (0.8647)^k \end{aligned}$$

The reliability of the old system at one year is 0.412 so we require k so that

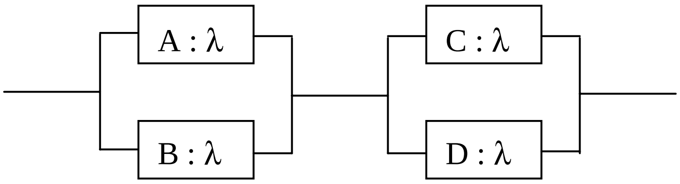
$$1 - (0.8647)^k \geq 0.412$$

or $$(0.8647)^k \leq 0.588$$

This can be solved explicitly, or by trial and error, giving k to be at least 4.

If the reliability at one year is to be $2 \times 0.412 = 0.824$ then we require k so that $1 - (0.8647)^k \geq 0.824$ leading to k having to be ≥ 12. Thus at least 12 components in parallel would be needed.

3. (i)



For each component $R(t) = e^{-\lambda t}$.

A/B subsystem has reliability $1 - (1 - e^{-\lambda t})(1 - e^{-\lambda t})$

$$= 2e^{-\lambda t} - e^{-2\lambda t}$$

C/D subsystem has the same reliability. Consequently, the reliability of the entire system is:

$$R_s(t) = (2e^{-\lambda t} - e^{-2\lambda t})^2 = 4e^{-2\lambda t} - 4e^{-3\lambda t} + e^{-4\lambda t}$$

(ii) Expected lifetime is

$$\int_0^\infty R_s(t)dt = \int (4e^{-2\lambda t} - 4e^{-3\lambda t} + e^{-4\lambda t})dt$$

$$= \left[\frac{4e^{-2\lambda t}}{-2\lambda} - \frac{4e^{-3\lambda t}}{-3\lambda} + \frac{e^{-4\lambda t}}{-4\lambda}\right]_0^\infty$$

$$= \frac{4}{2\lambda} - \frac{4}{3\lambda} + \frac{1}{4\lambda} = \frac{11}{12\lambda}$$

Result follows since the expected lifetime of each component is $\frac{1}{\lambda}$.

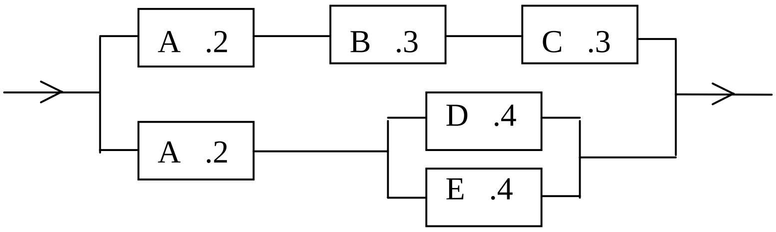
(iii) If $\lambda = 0.5$ per 1000 hours, mean lifetime is $\frac{11}{12(0.5)} = 1.833$ time units (i.e. 1833 hours).

Then $R_s(1.833) = 4e^{-1.833} - 4e^{-(1.5)(1.833)} + e^{-(2)(1.833)}$

$$= 0.6397 - 0.2558 + 0.0256$$

$$= 0.409$$

4. (i)



Since failure rates are constant, $R(t) = e^{-\lambda t}$ for each board.

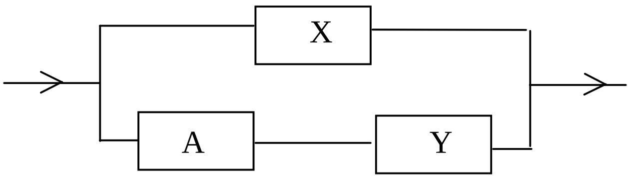
Replace ABC branch with component X with reliability

$$R_X(t) = e^{-(.2 + .3 + .3)t} = e^{-.8t}$$

Replace DE branch with component Y with reliability

$$R_Y(t) = 1 - [(1 - e^{-.4t})(1 - e^{-.4t})] = 2e^{-.4t} - e^{-.8t}$$

Now we have



Replace AY branch with component Z with reliability

$$R_Z(t) = e^{-.2t}(2e^{-.4t} - e^{-.8t})$$

$$= 2e^{-.6t} - e^{-t}$$

Finally, the reliability of system is

$$R_s(t) = 1 = [(1 - e^{-.8t})(1 - 2e^{-.6t} + e^{-t})]$$

$$= 1 - (1 - 2e^{-.6t} + e^{-t} - e^{-.8t} + 2e^{-1.4t} - e^{-1.8t})$$

Expected lifetime

$$E(L) = \int_0^\infty R_s(t)dt$$

$$= \left[\frac{2e^{-.6t}}{-.6} - \frac{e^{-t}}{-1} + \frac{e^{-.8t}}{-.8} - \frac{2e^{-1.4t}}{-1.4} + \frac{e^{-1.8t}}{-1.8}\right]_0^\infty$$

$$= \frac{2}{.6} - 1 + \frac{1}{.8} - \frac{2}{1.4} + \frac{1}{1.8} = 2.71 \text{ units.}$$

i.e. E(L) = 27,100 hours.

So the manufacturer's claim of at least 25,000 hrs is reasonable.