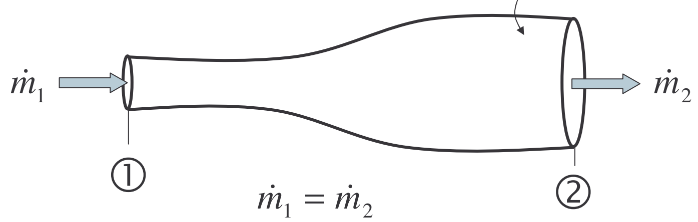
FLUID FLOW PRINCIPLES

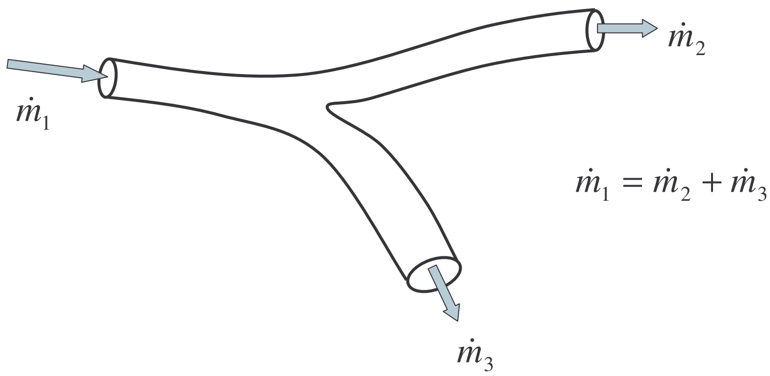
There are three basic principles that apply to fluid dynamics:

1. **Continuity**
2. **Momentum**
3. **Energy**

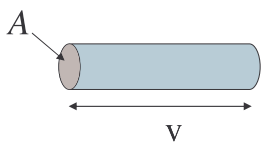
CONTINUITY

The principle of continuity is essentially that mass cannot be created or destroyed. In a steady fluid flow situation it means that the sum of the mass flows **in** to a control volume equal the sum of the mass flows **out** of the same control volume.





If the average flow velocity across a section '***A***' is '**v**', and the fluid density is **ρ** , then:



$$\text{volume/second} = A\text{v}$$

$$\therefore \text{mass/second} = \rho A\text{v}$$

$$\boxed{\dot{m} = \rho A\,\text{v}}$$

This principle of continuity can also be expressed in the form of Kirchoff's Law, which states that the sum of the mass flows in to a junction must be zero.
(Mass flow **in** is regarded as positive, mass flow **out** as negative)

$$\Sigma \dot{m}_{junction} = 0$$

For **incompressible** fluids (liquids) the density remains essentially constant.
The continuity equation can therefore be written in terms of the **volume** flow rates:

$$\Sigma \dot{V}_{junction} = 0$$

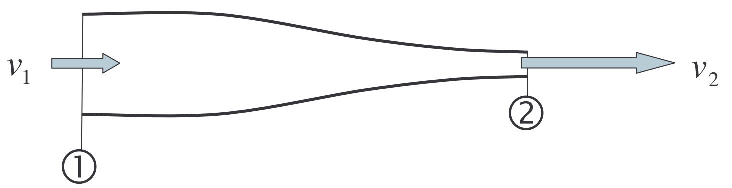
For **compressible** fluids (gases) density depends on pressure, temperature and ***R***, and these will need to be taken into account.

A useful form of the perfect gas equation is:

$$p = \rho RT \quad \text{or} \quad \rho = \frac{p}{RT}$$

Example 1:

In order to obtain a high speed jet of water (a fireman's nozzle) the diameter of the delivery pipe has to be reduced. If the normal flow rate of water from a fire pump is 20 kg/s what must the nozzle diameter be to obtain a water jet with a velocity of 45 m/s? (~ jet velocity needed for the water to reach the top of a 100m high building)



$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

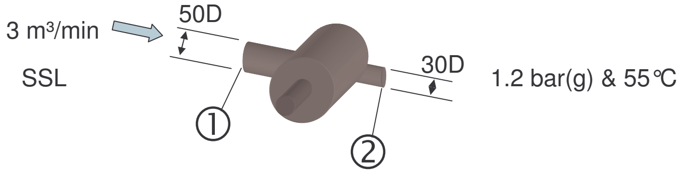
$$\dot{m} = \rho A_2 v_2$$

$$20 = 1000 \times \frac{\pi}{4} d_2^{\,2} \times 45$$

$$d_2 = \sqrt{\frac{4 \times 20}{\pi \times 45000}} = 0.02379 \text{ m} \quad \text{(or 23.8 mm)}$$

Example 2:

A rotary compressor induces 3 m³/min of air at SSL conditions through a 50mm diameter pipe and discharges the air at a pressure of 1.2 bar (gauge) and 55°C through a 30mm diameter pipe. What are the inlet and discharge air velocities?



At (1) we can write :

$$\dot{V}_1 = A_1 v_1$$

$$\frac{3}{60} = \frac{\pi}{4}(50\times10^{-3})^2 \times v_1$$

$$v_1 = 25.5\ m/s$$

At (2) we can write $\dot{m} = \rho_2 A_2 v_2$

$$\dot{m}_1 = \dot{m}_2 = \underbrace{\frac{p_1}{RT_1}}_{\rho_1}\dot{V}_1 = \frac{101.325\times\frac{3}{60}}{0.278\times(15+273)} = 0.06327\ \text{kg/s}$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{120+101.325}{0.278\times(55+273)} = 2.427\ \text{kg/m}^3$$

$$0.06327 = 2.427\frac{\pi}{4}(30\times10^{-3})^2 v_2$$

$$v_2 = 36.9\ \text{m/s}$$

MOMENTUM

Momentum is the application of Newton's Laws to fluid flow.

Newton's Second law states:

The force on a body equals its rate of change of momentum.

Momentum is a vector quantity and is defined as ***mass x velocity***

The rate of change of momentum is therefore given by: $\frac{d}{dt}(mv)$

There are typically two possibilities in the use of this equation:

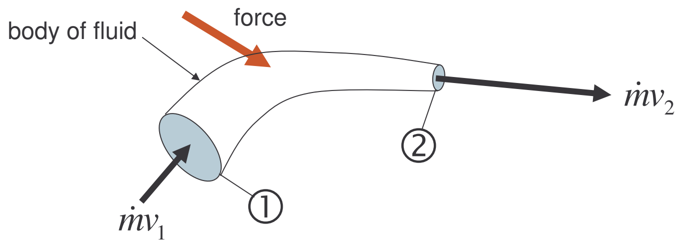
1. The mass of a body remains constant and its velocity changes; or
2. The velocities of a body of mass remain constant and the rate of mass changes.

The first is that often encountered in solid mechanics and it gives rise to the familiar version of Newton's Law: ***Force = mass x acceleration***

$$\frac{d}{dt}(mv) = m\frac{dv}{dt} = ma$$

The second is that often encountered in fluid dynamics:

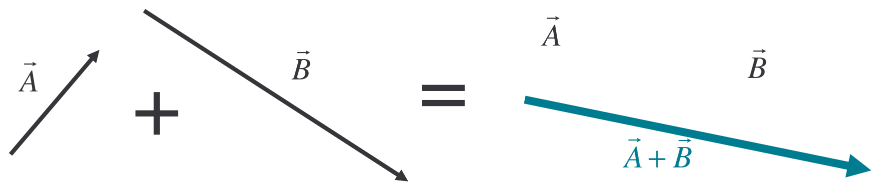
$$\frac{d}{dt}(mv) = v\frac{dm}{dt} = \dot{m}v$$



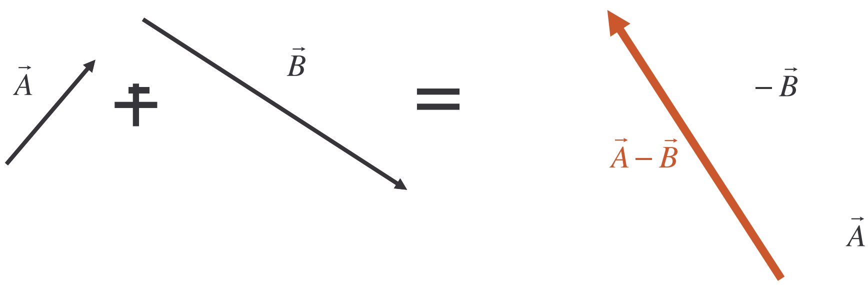
$$\begin{aligned}\text{resultant force} &= \text{rate of change of momentum}\\ &= \text{final momentum - initial momentum}\\ &= \dot{m}v_2 - \dot{m}v_1\end{aligned}$$

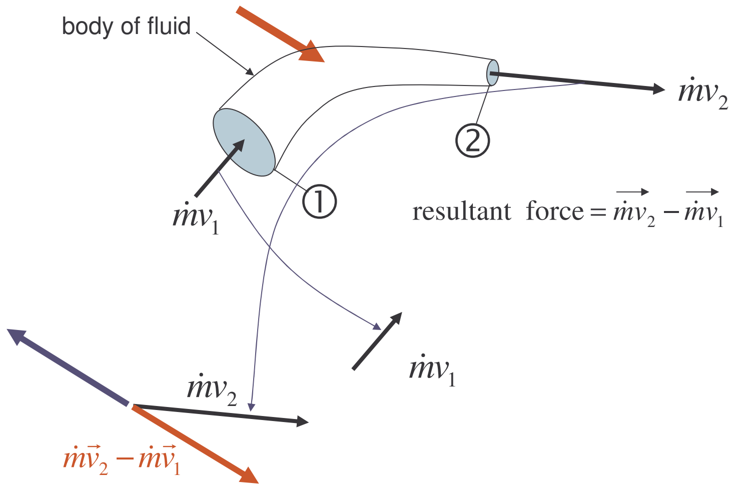
Because $\dot{m}v_2$ & $\dot{m}v_1$ are vector quantities it is the **vector difference** that is required to determine the force.

Vector sum: $\vec{A} + \vec{B}$



Vector difference: Reverse the vector and add. $\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) = \vec{A} + \overleftarrow{B}$





This is the force applied to the fluid to make it change direction and speed.

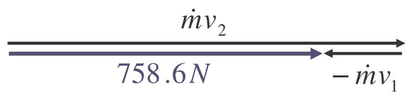
There is an equal and opposite force <u>on the duct</u> through which the fluid flows.
(Newton's Third Law)

This represents the essential principle of fluid machinery (turbines or pumps).

Example 3:

What force acts on the fireman's nozzle of Example 1 assuming the inlet diameter is 60mm? The flow rate from the pump is 20 kg/s and the water jet velocity is 45 m/s.

$$\dot{m}v_2 = 20 \times 45 = 900\,N$$
$$\dot{m}v_1 = 20 \times 7.07 = 141.4\,N$$



$$\dot{m} = \rho A_1 v_1$$

$$20 = 1000 \times \frac{\pi}{4}(60 \times 10^{-3})^2 \times v_1$$

$$v_1 = \frac{4 \times 20}{1000 \times \pi \times (60 \times 10^{-3})^2} = 7.07 \text{ m/s}$$

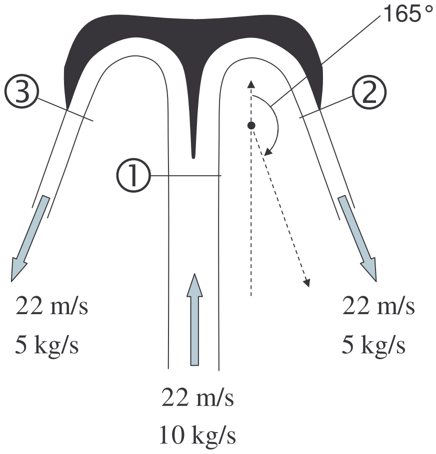
This force is the force on the water.

The force on the nozzle is equal and opposite.

This force has to be reacted by the person holding the nozzle:
it is close to the average weight of a man (77kg)!

Example 4:

A water jet is directed on to a symmetrical deflector which splits it in two and turns each stream through 165°. If the jet mass flow is 10 kg/s and its velocity is 22 m/s what force is applied to the deflector? Assume the jet is not slowed down by the deflector (i.e. frictionless flow)

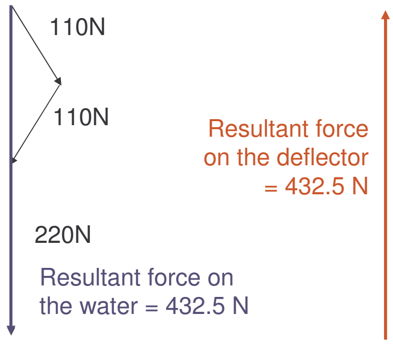


$$\dot{m}v_1 = 22\times10 = 220\text{N} \uparrow$$

$$\dot{m}v_2 = 22\times5 = 110\text{N} \searrow$$

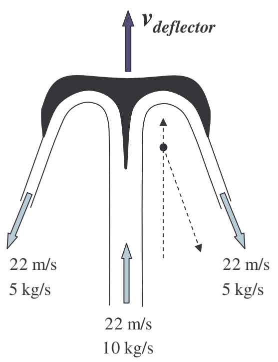
$$\dot{m}v_3 = 22\times5 = 110\text{N} \swarrow$$

$$\text{Resultant force} = \dot{m}\vec{v}_2 + \dot{m}\vec{v}_3 - \dot{m}\vec{v}_1$$



Example 5:

How does the force on the deflector vary if the deflector itself moves relative to the oncoming the jet?



The velocity of jet relative to the deflector = 22 - $v_{deflector}$

If the deflector speed = 22 m/s the speed of the jet relative to the deflector will be zero, and there will be no force applied to it.

It can be shown that this reduction in force is linear with deflector speed.

The power developed by the deflector = F x v

Maximum power output occurs when v = 11 m/s