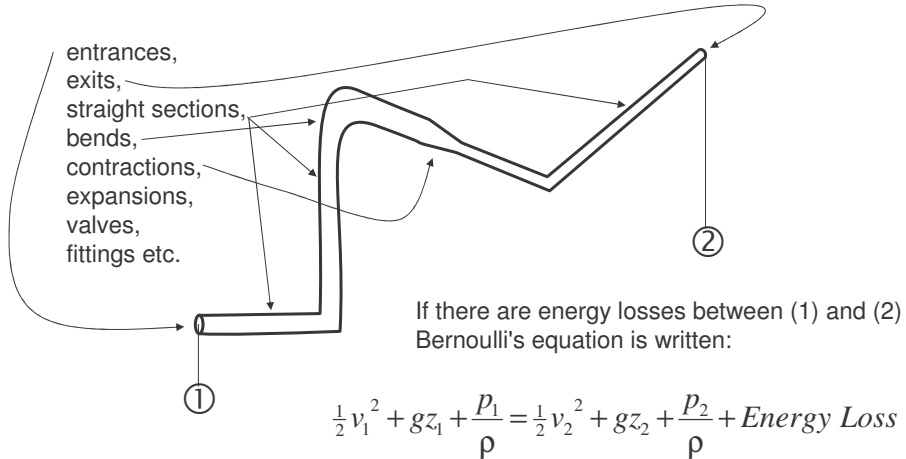


ENERGY LOSSES in FLUID FLOW

In practice there is no such thing as frictionless flow – and we must normally take frictional losses into account.

Frictional losses occur because of fluid viscosity and the creation of turbulence because of flow disturbances.

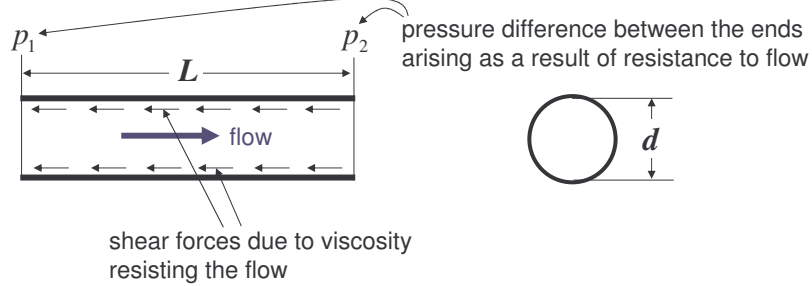


The usual form of the equation expressing these losses is to write it in terms of head – and the energy loss is expressed as a **head loss**.

$$\frac{v_1^2}{2g} + z_1 + \frac{p_1}{\rho g} = \frac{v_2^2}{2g} + z_2 + \frac{p_2}{\rho g} + \text{Head Loss}$$

$$\frac{v_1^2}{2g} + z_1 + \frac{p_1}{\rho g} = \frac{v_2^2}{2g} + z_2 + \frac{p_2}{\rho g} + H_L$$

Head loss because of fluid friction in straight length of pipe



The fluid contained in the pipe is not accelerating therefore the forces acting on it must sum to zero.

Pressure force = $(p_1 - p_2) \times \text{cross sectional area}$. i.e. $F_p = (p_1 - p_2) \times \frac{\pi}{4} d^2$

Total shear force = shear stress \times surface area. i.e. $F_s = \tau_w \times \pi d L$

$\tau_w = \text{shear stress at the pipe wall}$

Equating forces: $(p_1 - p_2) \times \frac{\pi}{4} d^2 = \tau_w \pi d L \quad \therefore (p_1 - p_2) = \frac{4L}{d} \tau_w$

The fluid shear stress acting at the wall is related to the **dynamic pressure** by what is known as the **friction factor**.

$$\tau_w = f \left(\frac{\rho v^2}{2} \right)$$

$$(p_1 - p_2) = 4f \frac{L}{d} \frac{\rho v^2}{2}$$

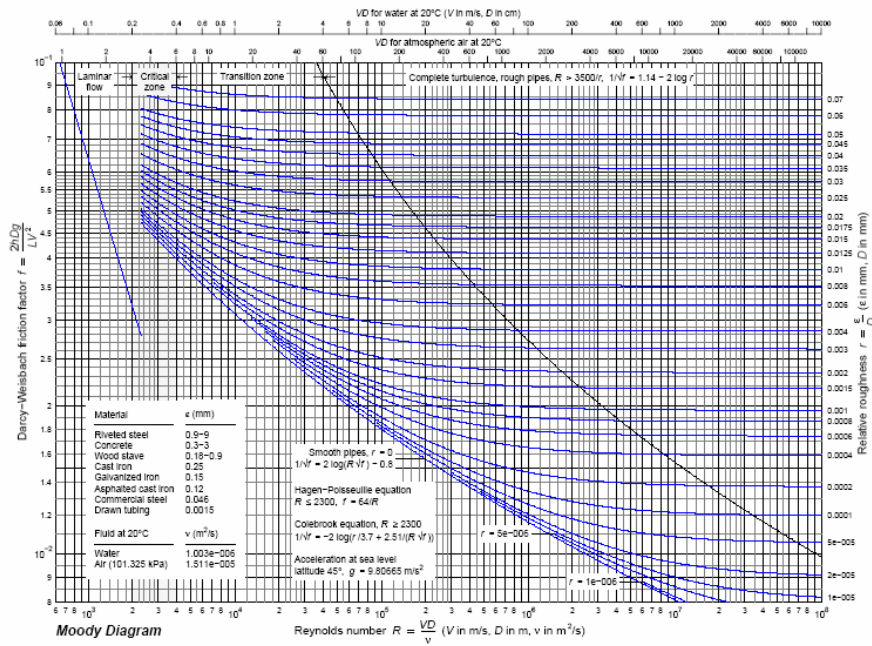
The head loss = $\frac{(p_1 - p_2)}{\rho g} = 4f \frac{L}{d} \frac{v^2}{2g}$

$$h_L = 4f \frac{L}{d} \frac{v^2}{2g}$$

The friction factor itself is a function of Reynolds Number (containing viscosity) and the relative roughness of the pipe inner surface.

For laminar flow: $f = \frac{16}{\text{Re}}$

For turbulent flow we obtain friction factor from a **Moody Diagram**:



Caution: In some versions of the Moody Diagram (US practice) the '4' in the expression is incorporated into the friction factor. This is usually evident from the diagram itself.

In the diagram above: $f = \frac{2hDg}{LV^2} \quad \therefore h_L = f \frac{LV^2}{d \cdot 2g}$

For laminar flow: $f = \frac{64}{Re}$ the '4' is missing

$f_{British} = \frac{f_{American}}{4}$

This does not normally cause a problem – it's simply a case of using the correct head loss expression with the matching friction factor.

Head loss due to bends, elbows, fittings, etc.

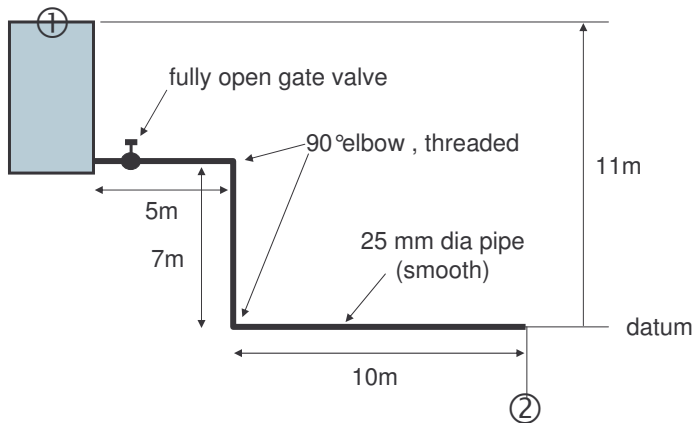
These are usually dealt with in one of two ways:

- Substitution of an equivalent length of straight pipe;
- Designation of a 'k' factor, where the head loss is given by: $h_L = k \frac{v^2}{2g}$

Fitting	k
Globe valve, fully open	10.00
Angle valve, fully open	2.00
Gate valve, fully open	0.15
Gate valve, 1/2 closed	2.10
Swing check valve, flow	2.00
Elbow 90° – flanged	0.30
Elbow 90° – threaded	1.50
Long radius 90°, flanged	0.20
Long radius 90°, threaded	0.70
Elbow 45°, threaded	0.40
Tee, Line flow, flanged	0.20
Tee, Line flow, threaded	0.90

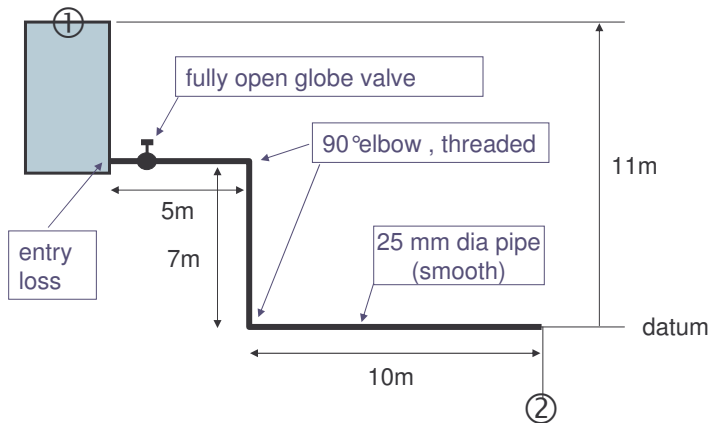
This information is often provided by manufacturers. However, some care is needed because some manufacturers use proprietary methods of head loss calculation.

Example: Determine the flow rate from a water tank with a pipe system as shown.



$$\frac{v_1^2}{2g} + z_1 + \frac{p_1}{\rho g} = \frac{v_2^2}{2g} + z_2 + \frac{p_2}{\rho g} + H_L$$

$$0 + 11 + \frac{p_{atm}}{\rho g} = \frac{v_2^2}{2g} + 0 + \frac{p_{atm}}{\rho g} + H_L \quad \therefore H_L = 11 - \frac{v_2^2}{2g}$$



$$H_L = \text{entry} + \text{valve} + \text{elbow} + \text{elbow} + (5+7+10)\text{straight pipe}$$

$$H_L = \frac{v^2}{2g} + (k_{\text{valve}} + k_{\text{elbow}} + k_{\text{elbow}}) \frac{v^2}{2g} + f \frac{L}{d} \frac{v^2}{2g} \quad (L=5+7+10)$$

However, before we can find f we need v !

We need the answer before we can solve the problem!

We proceed by estimating v , find the estimated head loss, and then check our estimate.

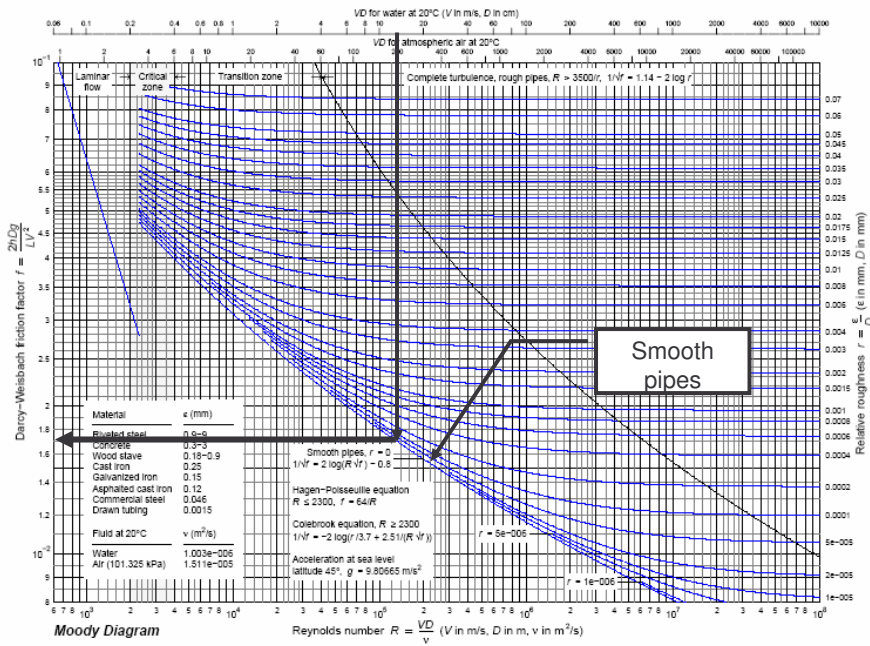
We can easily find v if there are no losses, and then reduce it by a 'reasonable' fraction.

$$H_L = 11 - \frac{v_2^2}{2g} \quad \text{If } H_L = 0 \quad \frac{v_2^2}{2g} = 11 \quad \therefore v = \sqrt{2 \times 9.81 \times 11} = 14.7 \text{ m/s}$$

Let's assume the actual velocity = $0.5 \times 14.7 \approx 7 \text{ m/s}$

$Vd = 7 \times 2.5 = 15$ from the Moody Diagram $f = 0.017$

↑
cm



$$H_{L_{estimate}} = \frac{v^2}{2g} + (k_{valve} + k_{elbow} + k_{elbow}) \frac{v^2}{2g} + f \frac{L}{d} \frac{v^2}{2g} (L = 5 + 7 + 10)$$

$$= \frac{7^2}{2 \times 9.81} \left\{ 1 + (0.15 + 1.5 + 1.5) + 0.017 \times \frac{22}{0.025} \right\} = 47.7 \text{ m}$$

Since $\frac{v_2^2}{2g} = 2.5$ this value exceeds the likely actual head loss by a factor of ~ 5

We need to reduce the velocity by dividing $\sim \sqrt{5}$ $\frac{7}{\sqrt{5}} \approx 3.1 \text{ m/s}$

Let's assume the actual velocity = 3.1 m/s

$$Vd = 3 \times 2.5 = 7.5 \text{ from the Moody Diagram } f = 0.020$$

↑
cm

$$\begin{aligned}
 H_{L_{estimate}} &= \frac{v^2}{2g} + (k_{valve} + k_{elbow} + k_{elbow}) \frac{v^2}{2g} + f \frac{L}{d} \frac{v^2}{2g} \quad (L = 5 + 7 + 10) \\
 &= \frac{3.1^2}{2 \times 9.81} \left\{ 1 + (0.15 + 1.5 + 1.5) + 0.02 \times \frac{22}{0.025} \right\} = 10.65 \text{ m} \\
 H_L &= 11 - \frac{v_2^2}{2g} = 11 - \frac{3.1^2}{2 \times 9.81} = 10.52 \text{ m}
 \end{aligned}$$

The estimated and actual values of the head loss are now almost equal.

$$\dot{m} = \rho A v = 1000 \times \frac{\pi}{4} 0.025^2 \times 3.1 = 1.52 \text{ kg/s}$$

This iterative method often occurs when analysing pipe flow systems. It is often readily carried out via a spreadsheet.