FLUID STATICS

Fluid statics can be regarded as a special case of fluid dynamics when the velocity of the fluid at all points is zero.

In Bernoulli's equation: $\frac{1}{2}v^2 + gz + \frac{p}{\rho} = const$ if $\boldsymbol{v} = 0$

$$gz + \frac{p}{\rho} = const \quad \text{or} \quad \boxed{p + \rho g z = const}$$

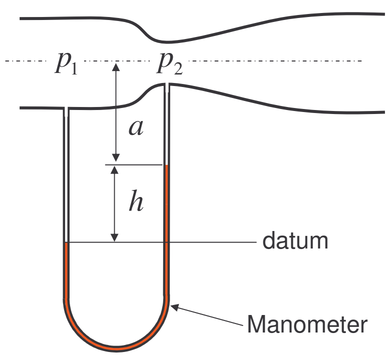
This is the fundamental equation of fluid statics.

It implies that:

- In a continuous body of static fluid the pressure at a particular level is everywhere the same;
- The pressure gradient is linear with depth.

It is these principles that enable manometry, and the determination of pressure forces on submerged surfaces.

The simple U-tube manometer



The pressure in the red fluid at the datum is the same in both arms of the manometer

$$p + \rho g z = const$$

$$p_1 + \rho g(h + a) = p_{datumLHS}$$

$$p_2 + \rho g a + \rho_M g h = p_{datumRHS}$$

$$\therefore p_1 + \rho g(h + a) = p_2 + \rho g a + \rho_M g h$$

$$p_1 - p_2 = (\rho_M - \rho) g h$$

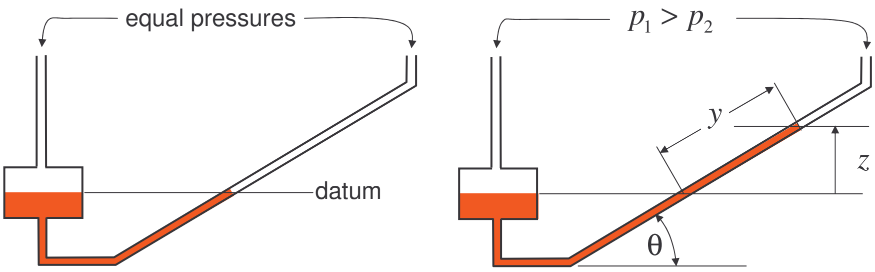
$$\frac{p_1 - p_2}{\rho} = (\frac{\rho_M}{\rho} - 1) g h$$

which gives us the result obtained earlier

If the fluid is a gas, and the manometer fluid is a liquid then typically: $\rho_M >> \rho$

$$p_1 - p_2 \cong \rho_M g h$$

The inclined manometer



Because the cross-section area of the reservoir is very much greater than the cross-section area of the tube, the liquid level in the reservoir changes very little in height when pressure is applied.

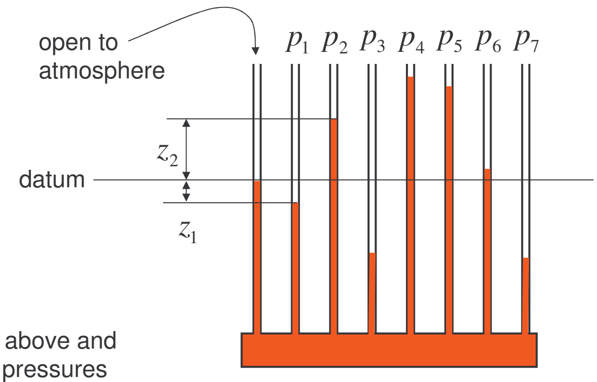
$$p_1 = p_{datumLHS} \qquad p_2 + \rho g z = p_{datumRHS} \qquad \frac{z}{y} = \sin\theta$$

$$p_1 = p_2 + \rho g y \sin\theta$$

$$p_1 - p_2 = \rho g y \sin\theta$$

By making the angle θ small the manometer can be made sensitive to small pressure changes.

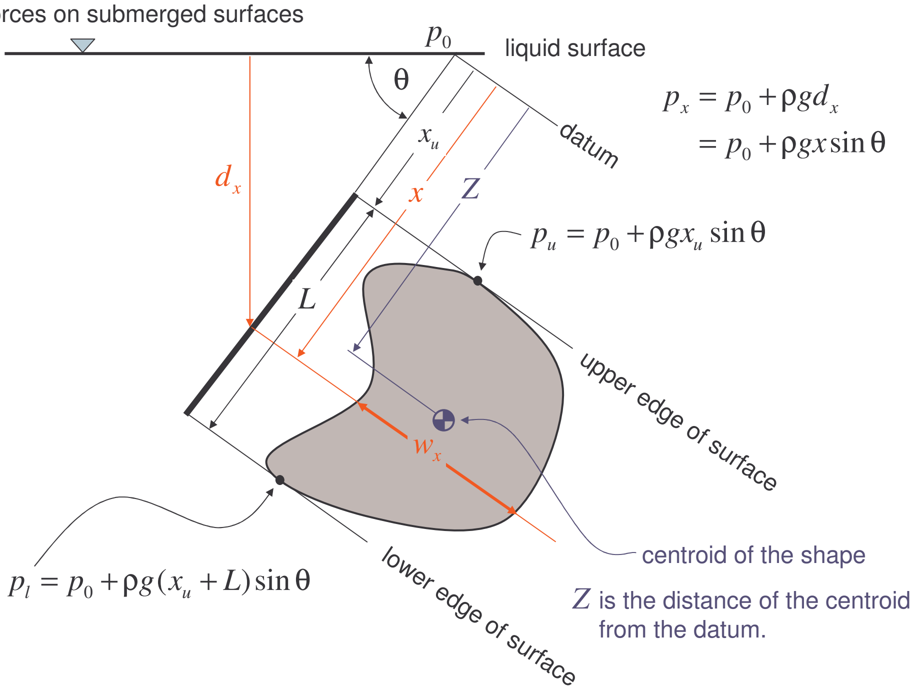
The multi-tube manometer



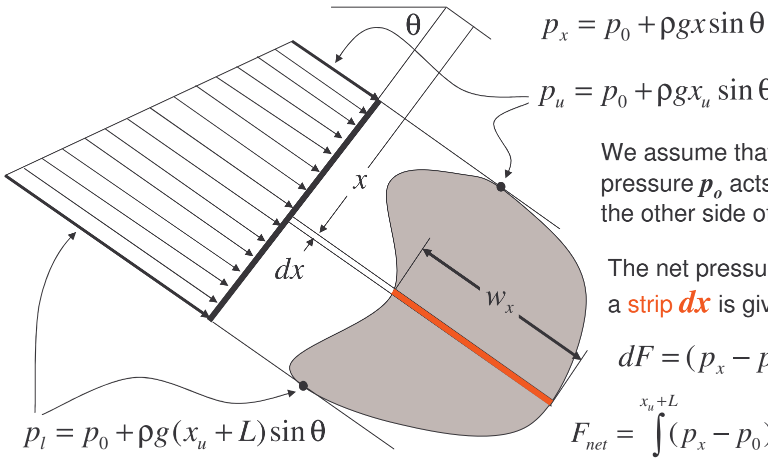
manometer liquid heights above and below the datum indicate pressures below and above atmospheric pressure

$$p_2 + \rho g z_2 = p_{atmos}$$

Forces on submerged surfaces



Forces on submerged surfaces



We assume that the surface pressure $\boldsymbol{p_o}$ acts uniformly on the other side of the shape

The net pressure force on a strip $\boldsymbol{dx}$ is given by:

$$dF = (p_x - p_0) w_x dx$$

$$F_{net} = \int_{x_u}^{x_u+L} (p_x - p_0) w_x dx$$

$$F_{net} = \int_{x_u}^{x_u+L} (p_0 + \rho g x \sin\theta - p_0) w_x dx$$

$$F_{net} = \rho g \sin\theta \int_{x_u}^{x_u+L} x w_x dx$$

$$F_{net} = \rho g \sin\theta \int_{x_u}^{x_u+L} x w_x dx$$

first moment of area about the datum

$$F_{net} = \rho g \sin\theta ZA$$

$$\boxed{F_{net} = \rho g ZA \sin\theta}$$

Point of action of the pressure force (centre of pressure)

To find the centre of pressure we need to take moments about the upper edge:

The net moment of the net pressure force on a strip dx is given by:

$$dM = (p_x - p_0) w_x x dx$$

$$M_{net} = \int_{x_u}^{x_u+L} (p_x - p_0) w_x x dx$$

$$M_{net} = \int_{x_u}^{x_u+L} (p_0 + \rho g \sin\theta x - p_0) w_x x dx$$

$$M_{net} = \rho g \sin\theta \int_{x_u}^{x_u+L} w_x x^2 dx$$

second moment of area about the datum I_{datum}

$$M_{net} = \rho g \sin\theta \int_{x_u}^{x_u+L} w_x x^2 dx = \rho g \sin\theta I_{datum}$$

$$\textit{from the parallel axis theorem } I_{datum} = I_c + AZ^2$$

$$M_{net} = \rho g \sin\theta (I_c + AZ^2)$$

This moment must equal the total net force x the distance to its point of application from the datum ***h***

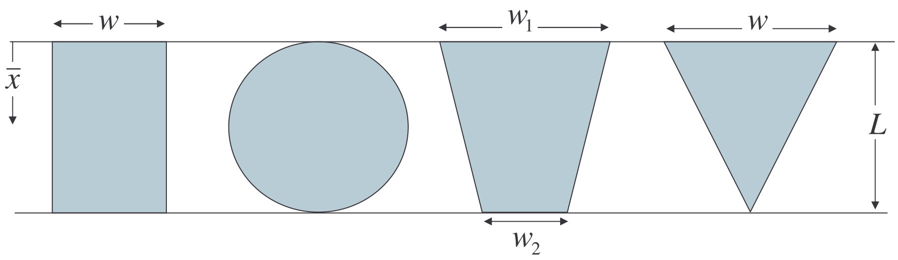
$$F_{net} h = \rho g \sin\theta (I_c + AZ^2)$$

$$h = \frac{\rho g \sin\theta (I_c + AZ^2)}{F_{net}} = \frac{\rho g \sin\theta (I_c + AZ^2)}{\rho g A Z \sin\theta}$$

$$h = \frac{I_c + AZ^2}{AZ} = \frac{I_c}{AZ} + Z \quad \text{or} \quad h - Z = \frac{I_c}{AZ}$$

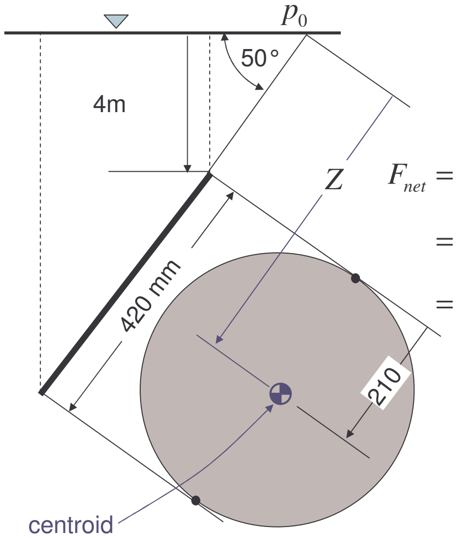
i.e. the pressure force acts below the centroid by a distance given by: $\frac{I_c}{AZ}$

It follows that if we know the **area**, **position of the centroid**, and **second moment of area about the centroid**, then we can find the pressure force on a surface, and its point of application.



A	wL	$\frac{\pi L^2}{4}$	$\frac{(w_1+w_2)L}{2}$	$\frac{wL}{2}$
$\bar{x}$	$\frac{L}{2}$	$\frac{L}{2}$	$\frac{(w_1+2w_2)L}{3(w_1+w_2)}$	$\frac{L}{3}$
I_x	$\frac{wL^3}{12}$	$\frac{\pi L^4}{64}$	$\frac{(w_1^2+4w_1w_2+w_2^2)L^3}{36(w_1+w_2)}$	$\frac{wL^3}{36}$

Example: Find the net pressure force and its point of application on a 420 mm diameter circular hatch cover at an angle of 50 degrees to the horizontal with its upper edge submerged 4m in sea water. (density = 1027 kg/m³)



$$F_{net} = \rho g Z A \sin\theta$$

$$Z = \frac{4}{\sin 50} + 0.210 = 5.432\,\text{m}$$

$$F_{net} = \rho g Z A \sin\theta$$
$$= 1027 \times 9.81 \times 5.432 \times \frac{\pi}{4}(0.42)^2 \times \sin 50$$
$$= 5808\,\text{N} \quad \text{or } 5.81\,\text{kN}$$

the net pressure force acts below the centroid by a distance given by: $\frac{I_c}{AZ}$

$$I_c = \frac{\pi L^4}{64} = \frac{\pi(420\times10^{-3})^4}{64} = 0.00152745\,\text{m}^4$$

$$A = 0.138544\,\text{m}^2$$

$$Z = 5.432\,\text{m}$$

$$\frac{I_c}{AZ} = \frac{0.00152745}{0.138544 \times 5.432} = 0.00203\,\text{m}$$

i.e. the net pressure force acts below the centroid by a distance <u>2.0 mm</u>