

## APPLICATIONS OF THE NON-FLOW ENERGY EQUATION

The non-flow energy equation applies to **solids, liquids and gases** in a closed system:

$$Q + W = \Delta U$$

However, because **solids and liquids** are essentially incompressible, their change in volume on being heated/cooled or compressed is very small. It follows that the work transfer term  $W$  is insignificant in most instances.

For example: the volume of 1 kg of water at atmospheric pressure and 0°C increases from 1 litre to 1.044 litres at a temperature of 100°C. The work done is therefore:

$$\begin{aligned} p\Delta V &= 101.325 \times 10^3 \times (1.044 - 1.00) \times 10^{-3} \\ &= 4.46 \text{ J} \end{aligned}$$

The specific internal energy change is:

$$\begin{aligned} u_{100^\circ\text{C}} - u_{0^\circ\text{C}} &= 419000 - 0 \\ &= 419000 \text{ J/kg} \end{aligned}$$

The specific enthalpy change is:

$$\begin{aligned} h_{100^\circ\text{C}} - h_{0^\circ\text{C}} &= (419000 - 0) + 4.46 \\ &= 419004.46 \text{ J/kg} \end{aligned}$$
$$h = u + pv$$

It follows that the (average)  $c_v$  value for water (between 0° and 100°C) is given by:

$$\begin{aligned} u_{100^\circ\text{C}} - u_{0^\circ\text{C}} &= c_v \Delta T = 419000 \text{ J/kg} \\ \therefore c_v &= \frac{419000}{100} = 4190.000 \text{ J/kgK} \text{ or } 4.19 \text{ kJ/kgK} \end{aligned}$$

and the (average)  $c_p$  value for water (between 0° and 100°C) is given by:

$$\begin{aligned} h_{100^\circ\text{C}} - h_{0^\circ\text{C}} &= c_p \Delta T = 419004.46 \text{ J/kg} \\ \therefore c_p &= \frac{419004.46}{100} = 4190.045 \text{ J/kgK} \text{ or } 4.19 \text{ kJ/kgK} \end{aligned}$$

i.e. for liquids and solids:  $u \approx h$  and

$$c_p \approx c_v = c \quad \text{where } c \text{ is virtually independent of pressure.}$$

Note however, that when a liquid changes phase to a vapour there is a very significant volume change and  $u$  and  $h$  can be very different!

Heating or cooling of a liquid or solid in a closed system occurs essentially at constant volume unaffected by the pressure.

**True constant volume** heating of a liquid or solid is close to a physical impossibility because of the enormous pressures that would be required.

---

## **gases**

The NFEE can be applied to the design of Internal Combustion engines, and any other device that can be described as 'positive displacement'. i.e. traps gas in a fixed or variable volume, and moves it from one location to another. Many compressors act in this way.

## **DESIGN BRIEF**

Design a four-cylinder petrol engine which will produce 60 kW of shaft power at an engine speed of 3000 RPM. Assume the engine operates on the 4-stroke cycle, and operates according to the ideal Otto cycle using air as the working fluid.

(Just over 80 bhp in old units!)

In this case, by **design** we mean to find the bore, stroke and combustion chamber volume that will provide the power specified.

As with all design, we need to make certain assumptions in order to proceed. If our assumptions are reasonable the design will be OK. If our assumptions need to be changed (to be more realistic) the design will alter.

### **We shall assume:**

Ideal Otto cycle – reversible adiabatic compression; isochoric heat transfer in; adiabatic expansion; isochoric heat transfer out.

Maximum cycle temperature: 2500 °C

Volume compression ratio: 9:1

A frictionless engine (!)

In order to produce 60kW at 3000 RPM the **work output/cylinder/cycle** is given by:

Power output = **work output/cylinder/cycle** × (number of cylinders) × power cycles/s

With a 4-stroke engine each cylinder produces a power cycle every 2 revolutions

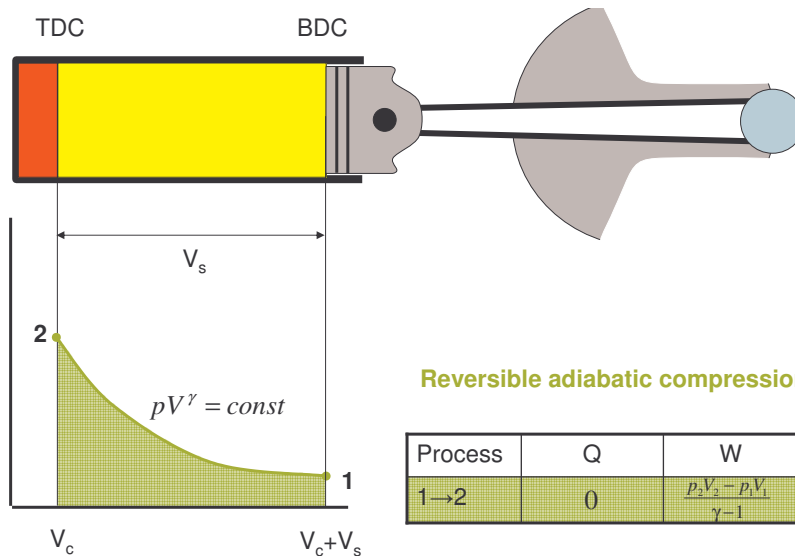
therefore power cycles/s =  $\frac{1}{2} \times \frac{RPM}{60}$

$$Power\ output = Work\ output\ / \ cylinder\ / \ cycle \times 4 \times \frac{1}{2} \frac{RPM}{60}$$

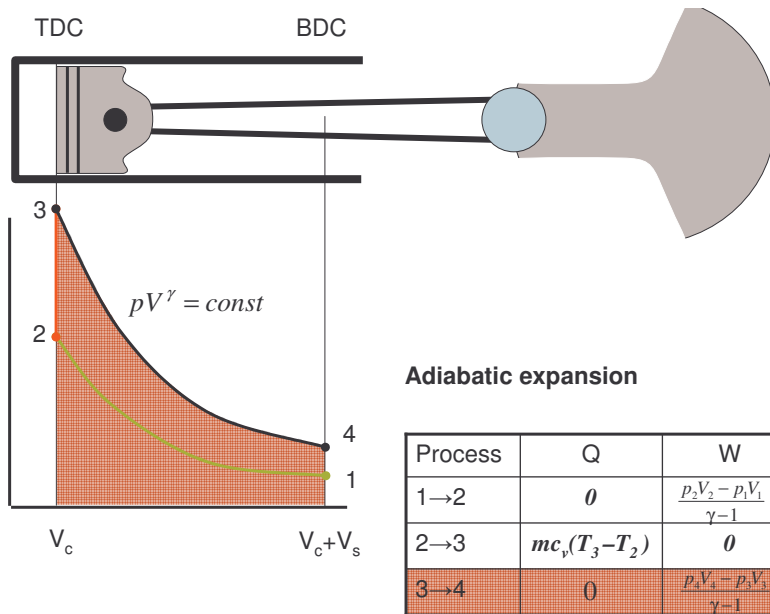
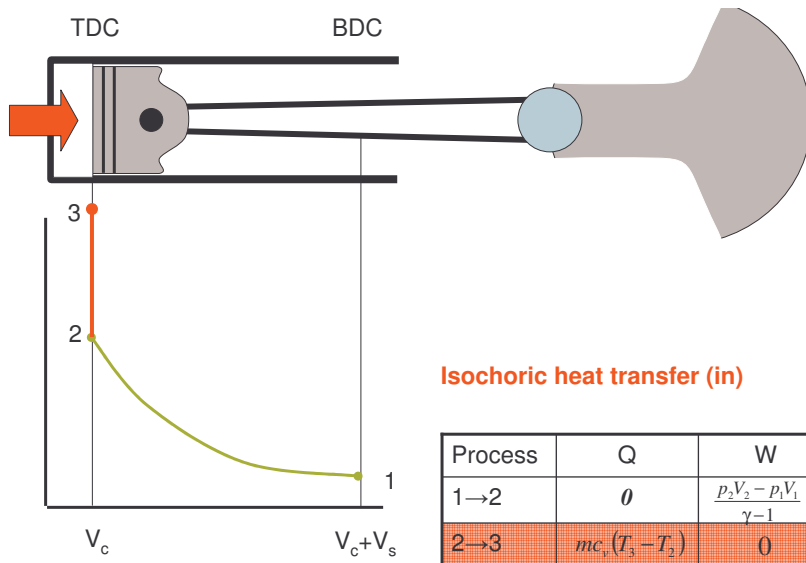
$$Work\ output\ / \ cylinder\ / \ cycle = \frac{Power\ output}{4 \times \frac{1}{2} \frac{RPM}{60}}$$

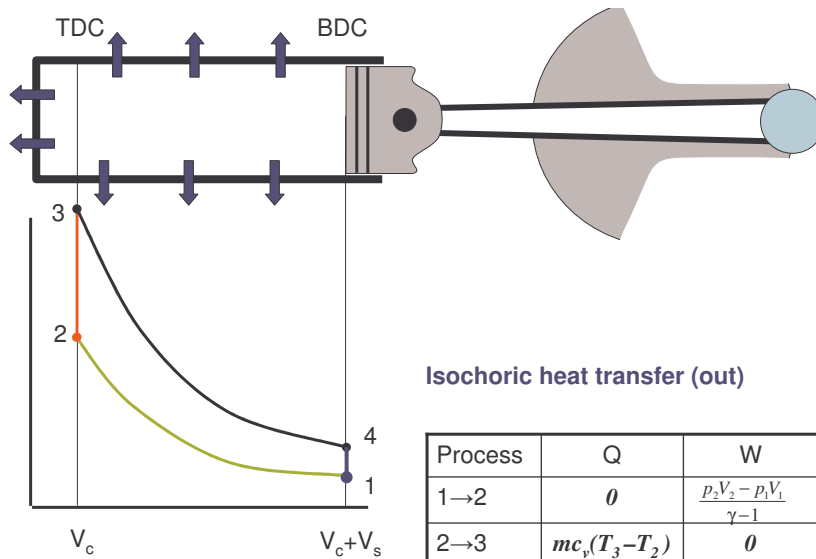
$$W_{net/cycle} = \frac{60kW}{4 \times \frac{1}{2} \frac{3000}{60}} = 0.6\ kJ / cycle$$

i.e. we must have a cylinder of sufficient size to produce **-0.6 kJ** of work every cycle (the negative sign is needed because it is work **out** of the cycle)



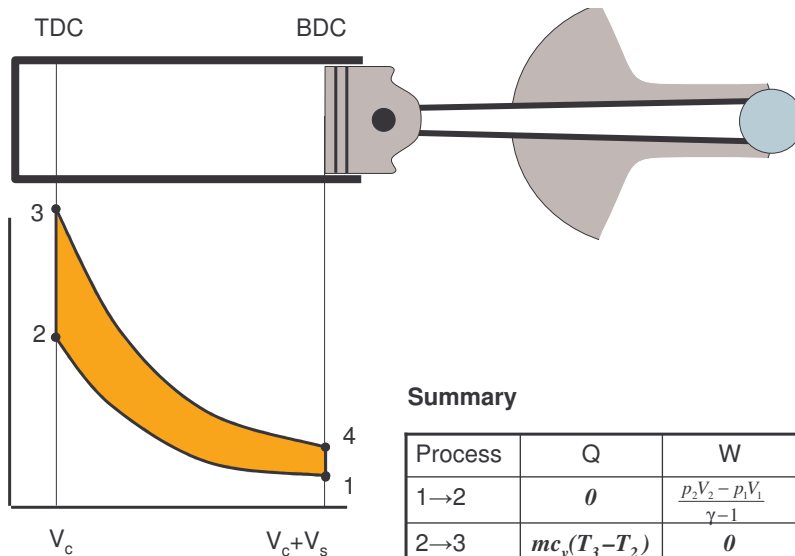
$$Volume\ compression\ ratio\ (r_v) = \frac{Cylinder\ volume\ at\ BDC}{Cylinder\ volume\ at\ TDC} = \frac{V_c + V_s}{V_c}$$





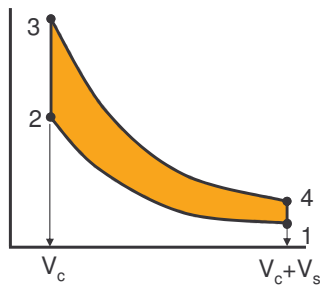
**Isochoric heat transfer (out)**

Process	Q	W	$\Delta U$
1→2	0	$\frac{p_2V_2 - p_1V_1}{\gamma - 1}$	$mc_v\Delta T$
2→3	$mc_v(T_3 - T_2)$	0	$mc_v\Delta T$
3→4	0	$\frac{p_4V_4 - p_3V_3}{\gamma - 1}$	$mc_v\Delta T$
4→1	$mc_v(T_1 - T_4)$	0	$mc_v\Delta T$



**Summary**

Process	Q	W	$\Delta U$
1→2	0	$\frac{p_2V_2 - p_1V_1}{\gamma - 1}$	$mc_v\Delta T$
2→3	$mc_v(T_3 - T_2)$	0	$mc_v\Delta T$
3→4	0	$\frac{p_4V_4 - p_3V_3}{\gamma - 1}$	$mc_v\Delta T$
4→1	$mc_v(T_1 - T_4)$	0	$mc_v\Delta T$
summation	<b>net heat transfer</b>	<b>net work transfer</b>	0



$$W_{net} = \frac{p_2 V_2 - p_1 V_1}{\gamma - 1} + \frac{p_4 V_4 - p_3 V_3}{\gamma - 1}$$

$$\text{but } V_2 = V_3 = V_c \quad \frac{V_c + V_s}{V_c} = r_v$$

$$\text{and } V_1 = V_4 = V_c + V_s$$

$$\therefore V_2 = V_3 = V_c = \frac{V_s}{r_v - 1} = \frac{V_s}{8} = 0.125V_s \text{ and}$$

$$V_1 = V_4 = V_c + V_s = \frac{r_v V_s}{r_v - 1} = \frac{9V_s}{8} = 1.125V_s$$

$$W_{net} = \frac{p_2 0.125V_s - p_1 1.125V_s}{\gamma - 1} + \frac{p_4 1.125V_s - p_3 0.125V_s}{\gamma - 1}$$

$$= (p_2 0.125 - p_1 1.125 + p_4 1.125 - p_3 0.125) \times \frac{V_s}{\gamma - 1}$$

We also need to find the pressures:

**For process 1 → 2:**

$$p_1 V_1^\gamma = p_2 V_2^\gamma \therefore p_2 = p_1 \left\{ \frac{V_1}{V_2} \right\}^\gamma = p_1 r_v^\gamma = 101.325 \times 8.5^{1.4} = 2027.23 \text{ kPa}$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \therefore T_2 = T_1 \frac{p_2 V_2}{p_1 V_1} = T_1 \frac{p_2}{p_1} \frac{1}{r_v} = (15 + 273) \frac{2027.23}{101.325} \frac{1}{8.5} = 677.9 \text{ K}$$

**For process 2 → 3:**

$$V_2 = V_3$$

$$\frac{p_2 V_2}{T_2} = \frac{p_3 V_3}{T_3} \therefore p_3 = p_2 \frac{T_3}{T_2} = 2027.23 \frac{(2500 + 273)}{677.9} = 8292.64 \text{ kPa}$$

**For process 3 → 4:**

$$p_3 V_3^\gamma = p_4 V_4^\gamma \therefore p_4 = p_3 \left\{ \frac{V_3}{V_4} \right\}^\gamma = p_3 \left\{ \frac{1}{r_v} \right\}^\gamma = 8292.64 \left\{ \frac{1}{8.5} \right\}^{1.4} = 414.48 \text{ kPa}$$

$$W_{net} = (p_2 0.125 - p_1 1.125 + p_4 1.125 - p_3 0.125) \times \frac{V_s}{\gamma - 1}$$

$$W_{net} = (2027.23 \times 0.125 - 101.325 \times 1.125 + 414.28 \times 1.125 - 8292.64 \times 0.125) \frac{V_s}{1.4 - 1}$$

and this value must equal = -0.6 kJ

$$(2027.23 \times 0.125 - 101.325 \times 1.125 + 414.28 \times 1.125 - 8292.64 \times 0.125) \frac{V_s}{1.4 - 1} = -0.6$$

$$V_s = \frac{-0.6 \times (1.4 - 1)}{2027.23 \times 0.125 - 101.325 \times 1.125 + 414.28 \times 1.125 - 8292.64 \times 0.125}$$

$$= 0.000557 \text{ m}^3 \text{ or } 0.557 \text{ litres}$$

The total engine swept volume (volumetric capacity) = 4 x 0.557 or a 2.227 litre engine

To find the bore and stroke of the engine we need to assume a bore: stroke ratio.  
In many engines this is ~1.0

$$0.000557 = \frac{\pi}{4} \text{ bore}^2 \times \text{stroke}$$

$$0.000557 = \frac{\pi}{4} \text{ bore}^3$$

$$\text{bore} = \sqrt[3]{\frac{0.000557 \times 4}{\pi}} = 0.0892 \text{ m or } 89.2 \text{ mm}$$

*You have just 'designed' your first IC petrol engine!*