APPLICATIONS OF THE STEADY FLOW ENERGY EQUATION

The steady flow energy equation applies to **liquids and gases** in a open system:

LIQUIDS

If we are dealing with a **liquid,** as we noted for the NFEE, there is very little difference in the enthalpy and internal energy – but the pressure may change significantly so we need to retain the **pv** term.

If we are pumping a **liquid** (continuous work transfer in), or using a **liquid** to power a turbine (continuous work transfer out) with no heat transfer to or from (heating or cooling) the liquid, then:

$$\dot{Q} + \dot{W} = \dot{m}\,\Delta[\tfrac{1}{2}v^2 + gz + u + p\mathrm{v}]$$

$$but\ \ \dot{Q} = 0 \ \text{ and } \Delta u = 0, \text{ so}$$

no temperature change

$$\dot{W} = \dot{m}\,\Delta[\tfrac{1}{2}v^2 + gz + p\mathrm{v}]$$

velocity specific volume

(We will study this equation in more detail later – under 'Fluid Flow')

If we are only heating or cooling a **liquid** (continuous heat transfer in/out) with no significant change in velocity, height or pressure, then:

$$\dot{Q} = \dot{m}\,\Delta[h] \quad = \dot{m}\ [h_2 - h_1] \ = \dot{m}c(T_2 - T_1)$$

(For a liquid $c_v = c_p = c$)

GASES

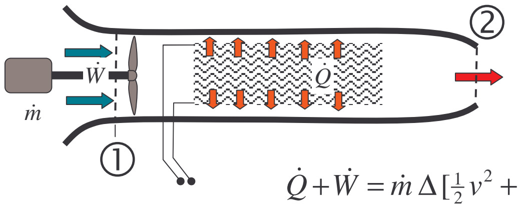
If we are only heating or cooling a **gas** (continuous heat transfer in/out) with no significant change in velocity, height or pressure, then:

$$\dot{Q} = \dot{m}\,\Delta[h] \quad = \dot{m}\ [h_2 - h_1] \ = \dot{m}c_p(T_2 - T_1)$$

Note that c_p must be used (if the pressure wasn't constant we wouldn't have a steady flow system!)

And also note that it is **mass** flow not volume flow required.

DESIGN BRIEF: Design a hot air gun which heats atmospheric air from 15℃ to 200℃. The single phase 240V mains limits the maximum power available to 3kW.
What air mass flow rate should be used?



T_1=15℃=288K

T_2=200℃=473K

$$\dot{Q}+\dot{W}=\dot{m}\,\Delta[\tfrac{1}{2}v^2+gz+h]$$

$$\text{in this case } v_1 \approx v_2, \text{ and } z_1 = z_2$$

$$\therefore \dot{Q}+\dot{W}=\dot{m}\,\Delta[h]=\dot{m}c_p(T_2-T_1)$$

$$\dot{Q}+\dot{W}=3kW$$

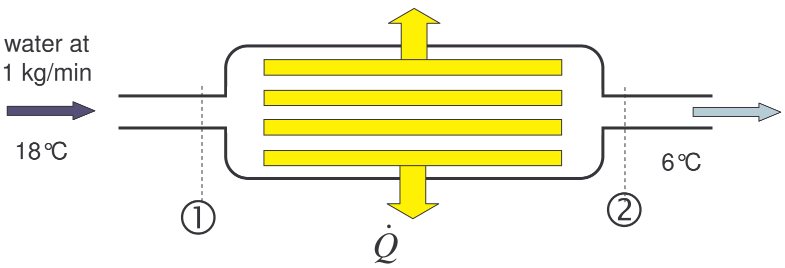
$$c_p \text{ for air} = 1.010\,kJ/kgK$$

$$\therefore 3.0=\dot{m}\times 1.010\times(473-288)$$

$$\dot{m}=\frac{3.0}{1.010\times(473-288)}=0.016\,kg/s$$

(~ 0.8 m^3 of air per minute at inlet conditions)

DESIGN BRIEF: A supply of chilled water at 6℃ is required to maintain the temperature of beer in a beer cellar. Find the cooling effect required to cool the water if the inlet temperature is 18℃ and the flow rate is 1 kg/min.



$$\dot{Q}+\dot{W}=\dot{m}\,\Delta[\tfrac{1}{2}v^2+gz+h]$$

$$\text{in this case } v_1 \approx v_2, \text{ and } z_1 = z_2 \;\&\; \dot{W}=0 \quad \therefore \dot{Q}=\dot{m}\,\Delta[h]=\dot{m}(h_2-h_1)$$

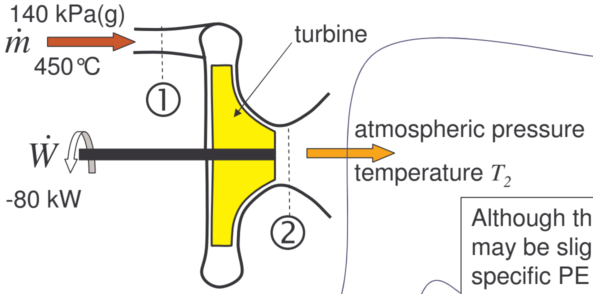
We can look up the specific enthalpy of water at 18℃ and 6℃ (Haywood p.8)

$$h_1 = 75.5\text{ kJ/kg (at 18°C)}$$
$$h_2 = 25.2\text{ kJ/kg (at 6°C)}$$

$$\therefore \quad \dot{Q}=\dot{m}\;[h_2-h_1]=\frac{1}{60}(25.2-75.5)\;=-0.84\text{ kW}$$

This heat transfer would probably be effected by the evaporation of a refrigerant in the yellow tubes. The refrigeration unit required would be comparatively small.

DESIGN BRIEF: A power turbine is supplied with hot high pressure air from a 'gas generator' at 140 kPa (gauge) and a temperature of 450℃. If the power turbine is required to produce 80 kW of shaft power output what hot air flow rate is required?



The inlet and exhaust velocities are likely to be very similar (by design):

Although the inlet and exhaust heights may be slightly different the change in specific PE compared with other energy changes is very very small. This is because the density of gases is usually low compared with liquids:

The hot gas passes through the turbine very quickly and therefore does not have time to lose heat to the surroundings:

$$\dot{Q}+\dot{W}=\dot{m}\,\Delta[\tfrac{1}{2}v^2+gz+h]$$

$$\text{in this case } v_1 \approx v_2, \text{ and } z_1 \approx z_2 \;\&\; \dot{Q}=0$$

$$\therefore \dot{W}=\dot{m}\,\Delta[h]=\dot{m}(h_2-h_1)=\dot{m}c_p(T_2-T_1)$$

We know the inlet temperature of the gas but we don't know the outlet (or exhaust) temperature, and we therefore have to make some assumptions as to exactly how the gas 'behaves' as it passes through the turbine.

Since the pressure is falling from 140 kPa to 100 kPa an **expansion** process is occurring. Since no heat is being lost the expansion process is **adiabatic**. Therefore the gas is undergoing an **adiabatic expansion**. We can therefore say that:

$$p\mathrm{v}^{\gamma}=const \quad \text{which defines the } \underline{\text{process}}$$

$$\text{and} \quad p\mathrm{v}=RT \quad \text{which relates the } \underline{\text{properties}}$$

$$p_1\mathrm{v}_1^{\gamma}=p_2\mathrm{v}_2^{\gamma} \longrightarrow \left(\frac{\mathrm{v}_1}{\mathrm{v}_2}\right)^{\gamma}=\frac{p_2}{p_1} \quad \therefore \quad \frac{\mathrm{v}_1}{\mathrm{v}_2}=\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$$

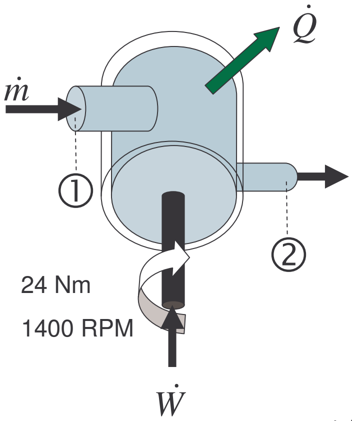
$$\frac{p_1\mathrm{v}_1}{T_1}=\frac{p_2\mathrm{v}_2}{T_2} \longrightarrow \frac{\mathrm{v}_1}{\mathrm{v}_2}=\frac{p_2}{p_1}\frac{\mathrm{T}_1}{\mathrm{T}_2}$$

$$\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}=\frac{p_2}{p_1}\frac{\mathrm{T}_1}{\mathrm{T}_2} \quad \text{or} \quad \frac{\mathrm{T}_2}{\mathrm{T}_1}=\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \frac{\mathrm{T}_2}{450+273}=\left(\frac{100}{140+100}\right)^{\frac{1.4-1}{1.4}} \quad \therefore \mathrm{T}_2=563K$$

$$\therefore \dot{W}=\dot{m}c_p(T_2-T_1) \quad \therefore -80\times10^3=\dot{m}\times1010\times(563-723)$$

$$\dot{m}=\frac{-80\times10^3}{1010\times(563-723)}=0.495\,\text{kg/s}$$

DESIGN BRIEF: A rotary vane compressor is designed to deliver 1.5 m³/min of air (measured at SSL conditions) at a pressure of 60 kPa (gauge). On test at 1400 RPM it was observed to require a driving torque of 24 Nm, and the air was delivered at a temperature of 90°C. How much cooling heat transfer is required?



$$\dot{Q}+\dot{W}=\dot{m}\,\Delta[\tfrac{1}{2}v^2+gz+h]$$

in this case $v_1 \approx v_2$, and $z_1 \approx z_2$ but $\dot{Q}\neq 0$

$$\therefore \dot{Q}+\dot{W}=\dot{m}\,\Delta[h]=\dot{m}(h_2-h_1)=\dot{m}c_p(T_2-T_1)$$

$$\dot{W}=torque\times rotational\ \ speed\,(radians/\sec)$$

$$=24\times\frac{1400\times 2\pi}{60}=3519\text{ watts}$$

$$\dot{m}=\frac{p\dot{V}}{RT}=\frac{101325\times\frac{1.5}{60}}{278\times 288}=0.0316\,\text{kg/s}$$

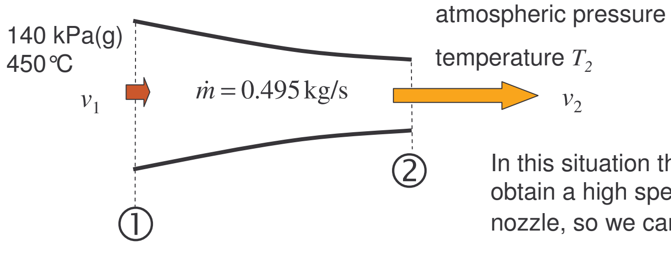
$$\therefore \dot{Q}+\dot{W}=\dot{m}c_p(T_2-T_1)$$

$$\dot{Q}+3519=0.0316\times 1010\times(90-15)$$

$$\dot{Q}=0.0316\times 1010\times(90-15)-3519=-1122\,\text{Watts}$$

(this might be provided by a water cooling jacket or by fins with a cooling air flow)

DESIGN BRIEF: It is decided to fit a thrust nozzle to the gas generator in the earlier example in order to design a small jet engine. The nozzle is supplied with hot high pressure air at 140 kPa (gauge) and a temperature of 450°C. How much thrust can be obtained for the air mass flow available (0.495 kg/s).



In this situation the whole idea is to obtain a high speed jet on exit from the nozzle, so we cannot assume $v_1 = v_2$

$$\dot{Q}+\dot{W}=\dot{m}\,\Delta[\tfrac{1}{2}v^2+gz+h]$$

in this case and $z_1 \approx z_2, \dot{Q}=0$ and $\dot{W}=0$

$$\therefore 0=\dot{m}\,\Delta[\tfrac{1}{2}v^2+h]$$

$$h_1-h_2=\tfrac{1}{2}({v_2}^2-{v_1}^2)\quad\text{or}\quad c_p(T_1\text{-}T_2)=\tfrac{1}{2}({v_2}^2-{v_1}^2)$$

Again, we know the inlet temperature of the gas but we don't know the outlet (or exit) temperature, and we therefore have to make some assumptions as to exactly how the gas 'behaves' as it passes through the nozzle.

Since the pressure is falling from 140 kPa to 100 kPa an **expansion** process is occurring. Since no heat is being lost the expansion process is **adiabatic**. Therefore the gas is undergoing an **adiabatic expansion**. We can therefore (exactly as before) say that:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

The calculation is identical therefore: $\frac{T_2}{450+273} = \left(\frac{100}{140+100}\right)^{\frac{1.4-1}{1.4}}$ $\therefore T_2 = 563K$

We can make one further reasonable assumption and that is that v_1 is likely to be very small compared with v_2, so we can effectively ignore: $\frac{1}{2}v_1^2$

$$c_p(T_1\text{-}T_2) = \tfrac{1}{2}v_2^2 \quad \text{or} \quad v_2 = \sqrt{2c_p(T_1\text{-}T_2)}$$

$$v_2 = \sqrt{2 \times 1010 \times (450+273-563)} = 568.5 \text{ m/s}$$

$$\text{Thrust} = \text{rate of change of momentum} = \dot{m}(v_2\text{-}v_1) = 0.495 \times (568.5 - 0) = 281.4N$$