

RADIATION

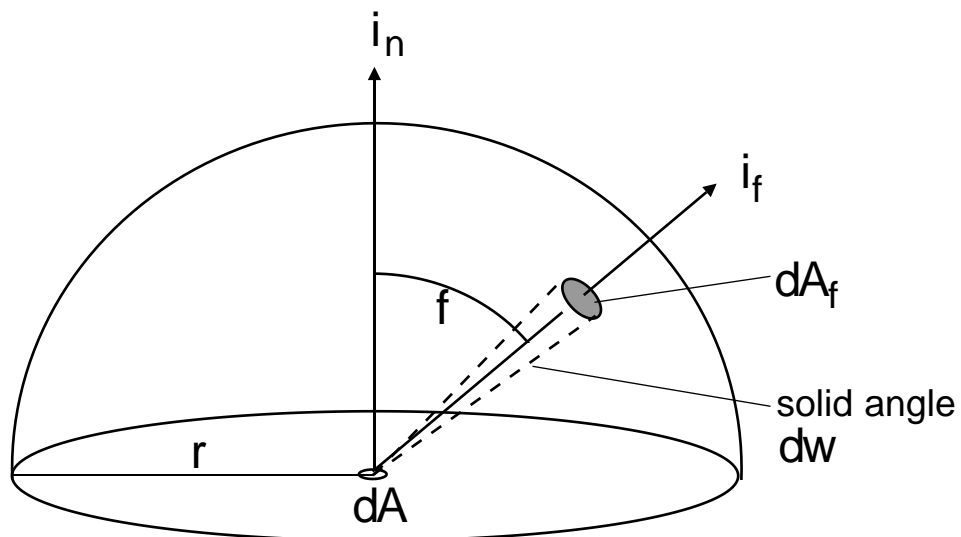
Our study so far has investigated the nature of infra-red radiation, and we derived an expression for the heat exchange between a grey body and completely black surroundings.

$$\dot{Q} = e s A (T_1^4 - T_2^4)$$

In order to study more general heat transfer problems than completely enclosed bodies it is necessary to know the spatial distribution of energy from a radiating point. It is also necessary to know the amount of radiation that a body can "see" of any other body (or of itself!).

INTENSITY of RADIATION

Consider a small area dA of emissive power \dot{E}'' at radius r . The energy emitted must all pass through a hemispherical



The intensity of radiation i_f in direction f is defined by:

$$i_f = \left(\frac{d\dot{E}''}{dw} \right)_f \quad (i)$$

dw is the small solid angle subtended by the small area of hemisphere dA_f at the small emitting area dA .

The value of dw is given by :

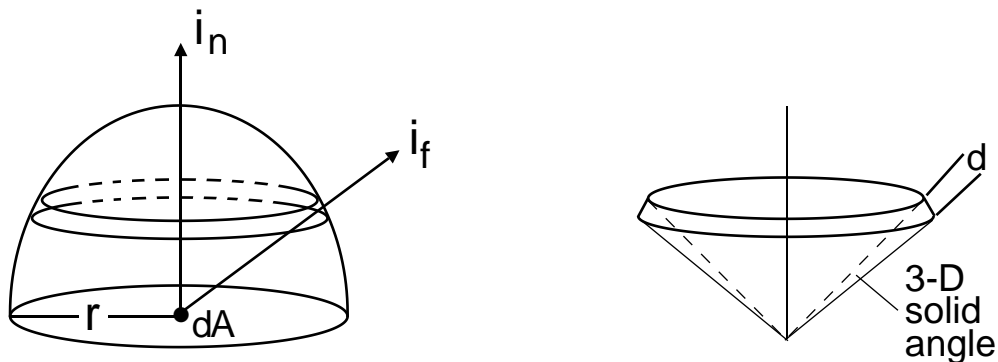
$$dw = \frac{dA_f}{r^2} \quad (ii)$$

Lambert's Law of diffuse radiation states that :

$$i_f = i_n \cos f \quad (iii)$$

where i_n is the *normal* intensity of radiation.

This may be determined for black and grey bodies.



The circumferential strip subtends the solid angle dw at

$$dw = \frac{(2\pi r \sin f) r df}{r^2}$$

ie $dw = 2\pi \sin f df$

Now from eqns (i) and (iii)

$$i_f = i_n \cos f = \left(\frac{d\dot{E}''}{dw} \right)_f$$

Hence $d\dot{E}''_f = i_n \cos f \cdot 2\pi \sin f \, df$

Integrating for the whole hemisphere :

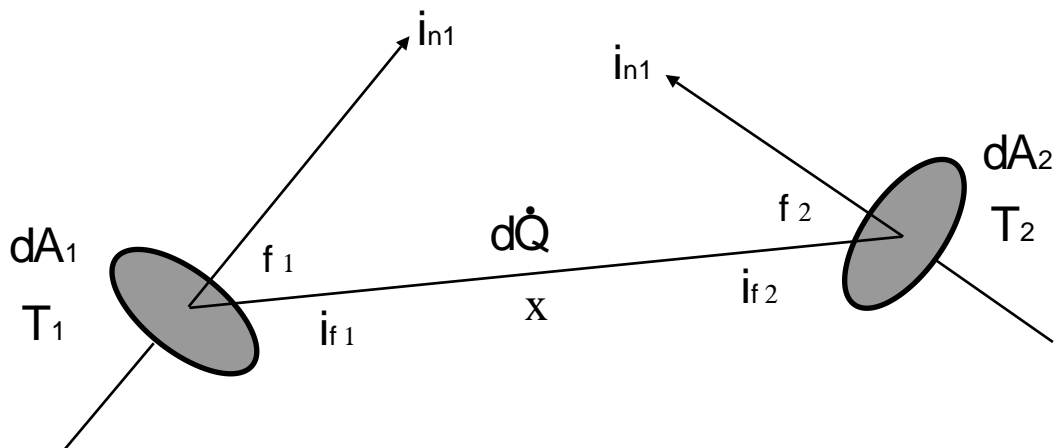
$$\dot{E}'' = \int_0^{\pi/2} i_n \cos f \cdot 2\pi \sin f \, df = \pi i_n$$

For a black surface; $\dot{E}'' = s T^4 = \pi i_n$; hence $i_n = \frac{s T^4}{\pi}$

For a grey surface; $\dot{E}'' = e s T^4 = \pi i_n$; hence $i_n = \frac{e s T^4}{\pi}$

THE GEOMETRIC FACTOR

If we have two arbitrarily disposed small black surfaces as shown below:



The heat transfer rate from surface 1 to surface 2 is given by " the rate of energy emitted by 1 intercepted & absorbed by 2, less the rate of energy emitted by 2 intercepted & absorbed by 1". ie:

$$\begin{aligned} d\dot{Q}_{12} &= d\dot{E}_{f_1}'' dA_1 - d\dot{E}_{f_2}'' dA_2 \\ &= i_{f_1} dw_1 dA_1 - i_{f_2} dw_2 dA_2 \\ &= i_{n1} \cos f_1 \left(\frac{dA_2 \cos f_2}{x^2} \right) dA_1 - i_{n2} \cos f_2 \left(\frac{dA_1 \cos f_1}{x^2} \right) dA_2 \end{aligned}$$

$$\text{but } i_{n1} = s T_1^4 / \rho \text{ and } i_{n2} = s T_2^4 / \rho$$

$$\text{therefore } d\dot{Q}_{12} = (T_1^4 - T_2^4) s \frac{\epsilon \cos f_1 \cos f_2 dA_1 dA_2}{\rho x^2}$$

For finite areas the expression must be integrated.

$$\text{ie } \dot{Q}_{12} = (T_1^4 - T_2^4) s \iint_{A_1 A_2} \frac{\cos \theta_1 \cos \theta_2 dA_1 dA_2}{r^2} \quad (\text{iv})$$

The solution to this integral is often complex.

We can simplify the situation by defining a GEOMETRIC FACTOR 'F' as the fraction of energy emitted by one surface that is intercepted by another.

$$\text{ie } \dot{Q}_{1 \rightarrow 2} = F_{12} s A_1 T_1^4$$

Fraction which lands on A₂
Total energy emitted by A₁

$$\text{similarly } \dot{Q}_{2 \rightarrow 1} = F_{21} s A_2 T_2^4$$

It follows that:

$$\dot{Q}_{12} = \dot{Q}_{1 \rightarrow 2} - \dot{Q}_{2 \rightarrow 1} = s (T_1^4 F_{12} A_1 - T_2^4 F_{21} A_2)$$

$$\text{from (iv) } \dot{Q}_{12} = s (T_1^4 - T_2^4) \cdot \text{const.}$$

It therefore follows that:

$$F_{12} A_1 = F_{21} A_2 = \iint_{A_1 A_2} \frac{\cos \theta_1 \cos \theta_2 dA_1 dA_2}{r^2}$$

$$\text{ie } \dot{Q}_{12} = A_1 F_{12} s (T_1^4 - T_2^4) = A_2 F_{21} s (T_1^4 - T_2^4)$$

Geometric factors may be found by analysis, analogy etc and are presented in tables or charts.

The charts are called HOTTTEL charts .

Geometric factors can be added or subtracted .

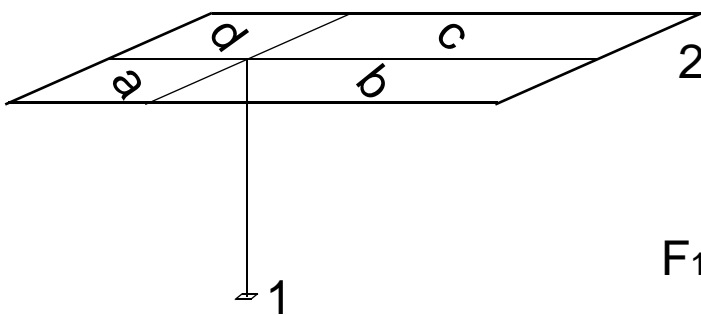
The sum of the factors from one body is 1 (unity).

For a three body system, if $F_{12} = 0.3$ then $F_{13} = 0.7$

In general for N surfaces there are N^2 factors, but there are only $N(N-1)/2$ independent factors.

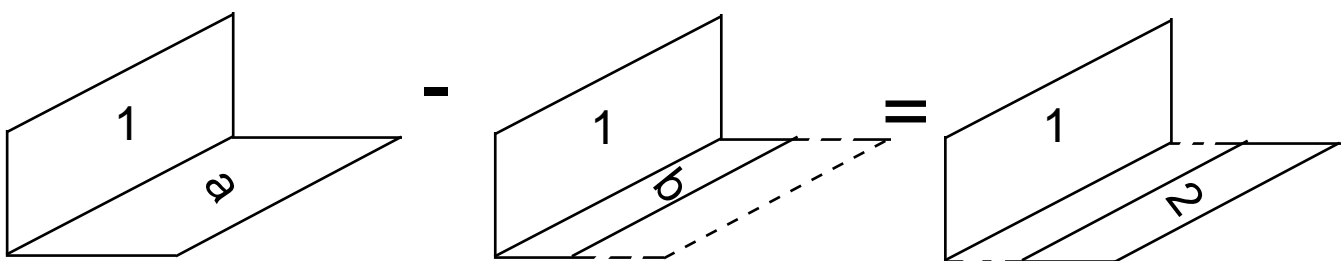
If bodies cannot 'see' themselves there are only $N(N-3)/2$ factors.

For example for small area '1' and large area '2'.



$$F_{12} = F_{1a} + F_{1b} + F_{1c} + F_{1d}$$

A chart is available which gives the Geometric Factor for a small area parallel to a large area with the small area located in line with one corner of the large area.



$$F_{12} = F_{1a} - F_{1b}$$

Use the chart for 2 planes at right angles with a common edge.

GREY BODY RADIATION

The Geometric Factor allows for the amount of radiation two surfaces can 'see' of each other, and although we developed it with black body surfaces it obviously applies to grey or any other type of surface.

However to solve grey body heat transfer problems we must allow for emissivity.

In all but the very simplest cases where the geometric factor is unity, or a simple ratio, so that the non-absorbed radiation is re-incident and re-reflected and a series may be formed and summed, the approach to grey body radiation heat transfer is through RADIOSITY.

We can set up an 'electrical' analogy for the radiation interchange between two BLACK bodies as we did previously:

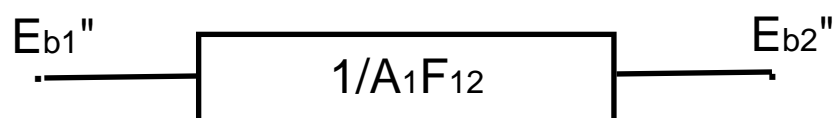
$$\text{ie } \dot{Q}_{12} = A_1 F_{12} s (T_1^4 - T_2^4)$$

\dot{Q}_{12} is analogous to current

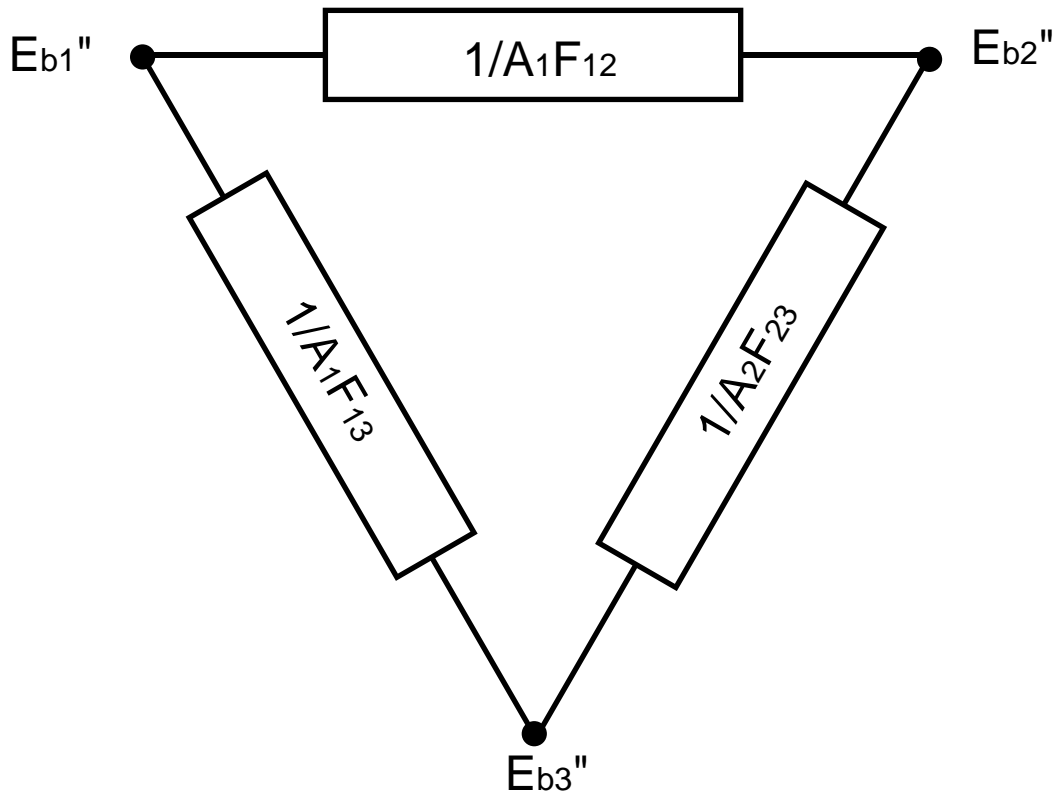
$s (T_1^4 - T_2^4)$ is analogous to voltage

ie $E_{b1}'' - E_{b2}''$ is analogous to voltage

therefore $\frac{1}{A_1 F_{12}}$ is analogous to resistance



For three BLACK surfaces which can 'see' each other:



$$\dot{Q}_{12} = \frac{\dot{E}_{b1''} - \dot{E}_{b2''}}{\frac{1}{A_1 F_{12}}} \quad \dot{Q}_{13} = \frac{\dot{E}_{b1''} - \dot{E}_{b3''}}{\frac{1}{A_1 F_{13}}} \quad \text{etc}$$

$$A_1 F_{12} = A_2 F_{21}; \quad F_{12} + F_{13} = 1$$

$$A_3 F_{31} = A_1 F_{13}; \quad F_{23} + F_{21} = 1$$

$$A_3 F_{32} = A_2 F_{23}; \quad F_{31} + F_{32} = 1$$

Use charts to find the appropriate GF's

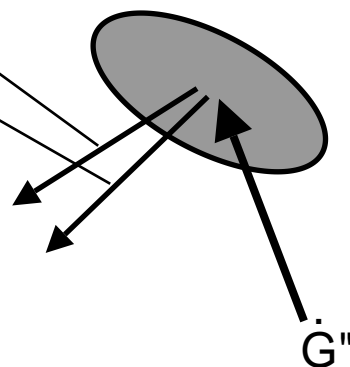
For GREY body radiation we need to allow for the emissivity effect.

We do this by replacing \dot{E}_b'' by \dot{J}'' , the RADIOSITY.

RADIOSITY

A grey body does not absorb all the incident radiation so that the **TOTAL** emitted energy from a grey body (termed RADIOSITY (\dot{J}'')) is given by:

$$\dot{J}'' = \underline{\dot{E}_g''} + r \underline{\dot{G}''} \quad (v)$$



G'' is the incident radiation per unit area

and $\dot{E}_g'' = e \dot{E}_b''$ the emitted radiation/unit area

Replacing E_b'' by J'' may be done by introducing another resistance into the electrical circuit to modify the potential.

The NET radiation leaving a grey surface area A_1 is given by:

$$A_1 (\dot{J}_1'' - \dot{G}_1'')$$

but
$$\dot{G}_1'' = \frac{\dot{J}_1'' - e \dot{E}_{b1}''}{r_1} \quad \text{from eqn. (v)}$$

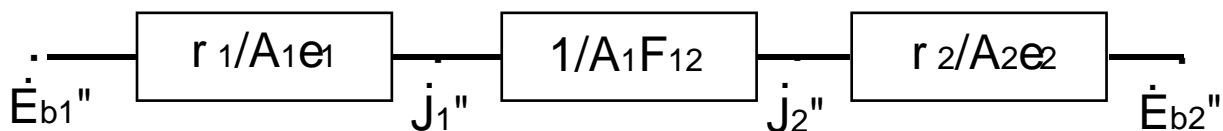
therefore Net Radiation =
$$A_1 \left(\dot{J}_1'' - \frac{\dot{J}_1'' - e \dot{E}_{b1}''}{r_1} \right)$$

for $t = 0, r_1 = 1 - e$

ie NET radiation =
$$\frac{A_1 e}{r_1} (\dot{E}_{b1}'' - \dot{J}_1'')$$

This can be incorporated into the electrical analogy by an

additional resistance of value :
$$\frac{r_1}{A_1 e}$$



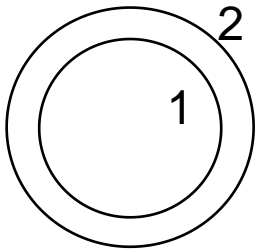
$$\dot{Q}_{12} = \frac{s (T_1^4 - T_2^4)}{\text{resistances}}$$

For the case of 'infinite' parallel flat plates $A_1 = A_2$

$$\dot{Q}_{12} = \frac{\epsilon A_1 (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

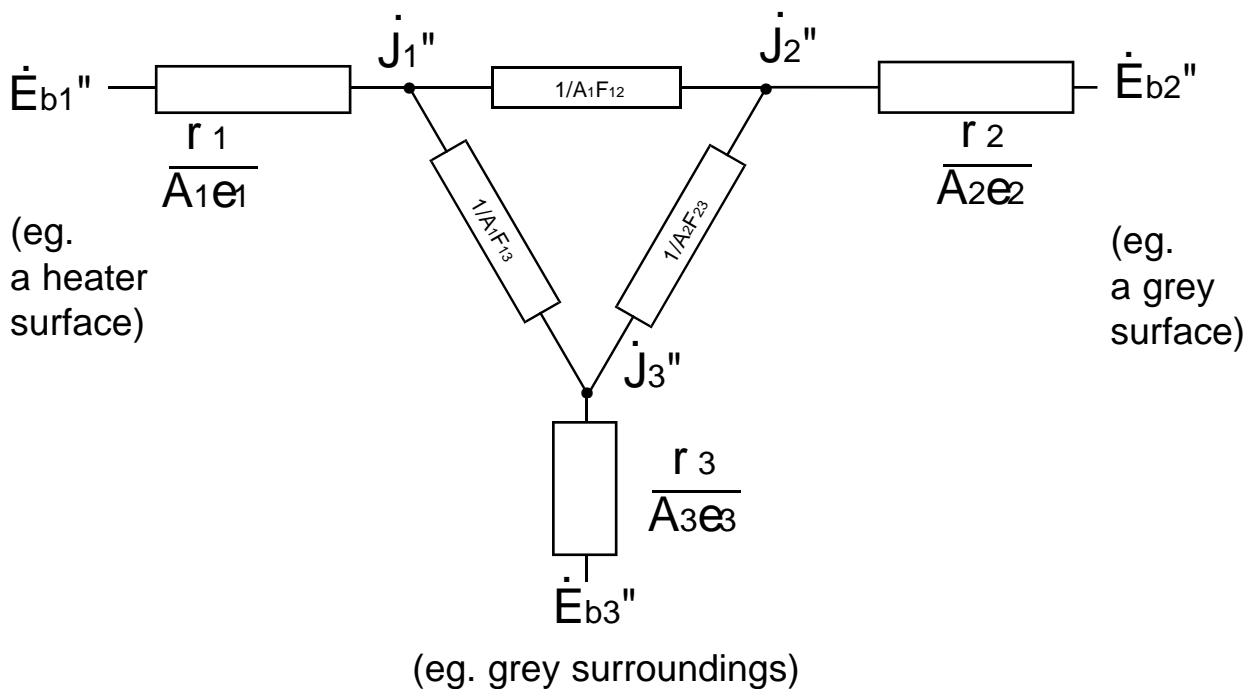
This can also be verified by summing a multiple reflection series since the geometric factor is unity .

For two long concentric cylinders it can be shown that:-



$$\dot{Q}_{12} = \frac{\epsilon A_1 (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{d_1}{d_2} \left(\frac{1}{\epsilon_2} - 1 \right)}$$

For three GREY surfaces which can 'see' each other:



We apply Kirchoff's Law to each J'' point

ie the sum of the current at each node = 0

$$\frac{E_{b1}'' - J_1''}{\frac{r_1}{A_1 \epsilon_1}} + \frac{J_2'' - J_1''}{A_1 F_{12}} + \frac{J_3'' - J_1''}{A_1 F_{13}} = 0 \text{ etc}$$

In this case we would obtain three equations which could be solved by Gaussian elimination, by hand or by computer.

For matrix formulation the equations may be re-written as:

$$J_1'' - (1 - \epsilon_1) [F_{12} J_2'' + F_{13} J_3''] = \epsilon_1 E_{b1}'' \quad \text{etc.}$$

or, in general, for the *ith* equation involving N surfaces

$$J_i'' - (1 - \epsilon_i) \sum_{j=1}^N F_{ij} J_j'' = \epsilon_i E_{bi}''$$

in which $F_{ii} = 0$ for the case $j=i$ in the summation unless the surface can 'see' itself.

When all the J'' values are found the heat transfer rates can be calculated.

Special Cases

(1) Black Surroundings : $J'' = E_b''$

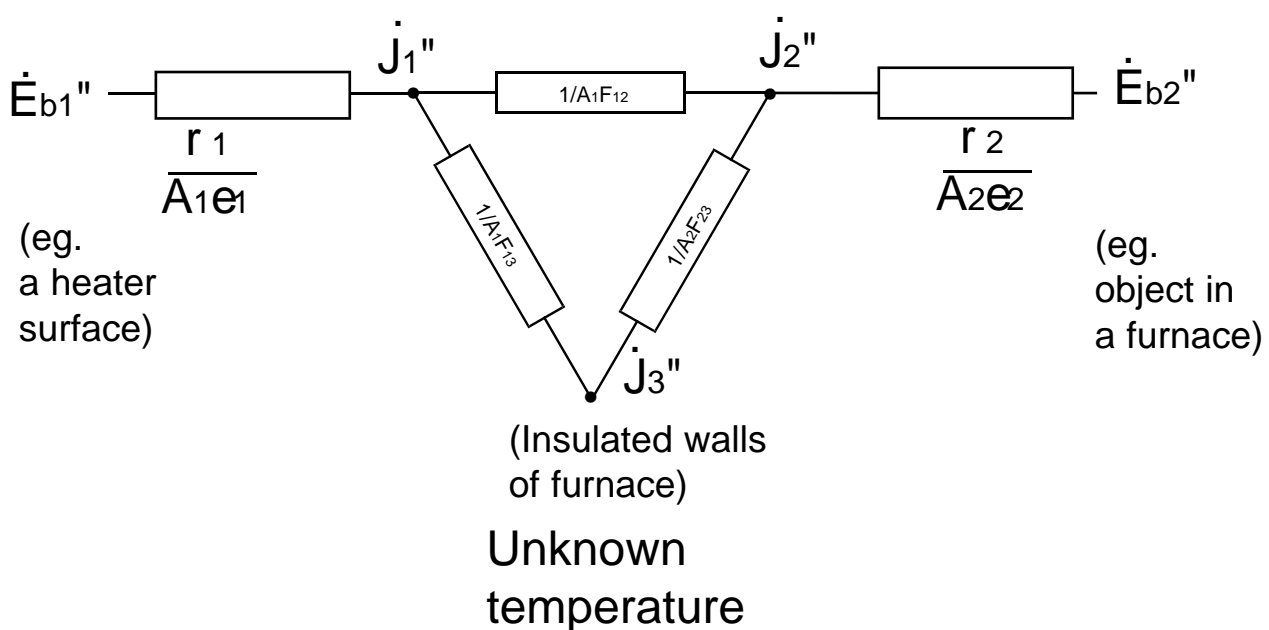
(2) Insulated or refractory surfaces

An 'insulated' or refractory surface in a radiation network has no net exchange of heat but influences a heat transfer process because it absorbs and re-radiates energy.

In this case the refractory body will have a J'' point representing the steady state temperature (potential) which it will achieve which must be included.

The temperature can be obtained from $J'' = s T^4$

In such a system (below) the total heat transfer from 1 to 2 is enhanced by the re-radiation from surface 3.



A series/parallel circuit

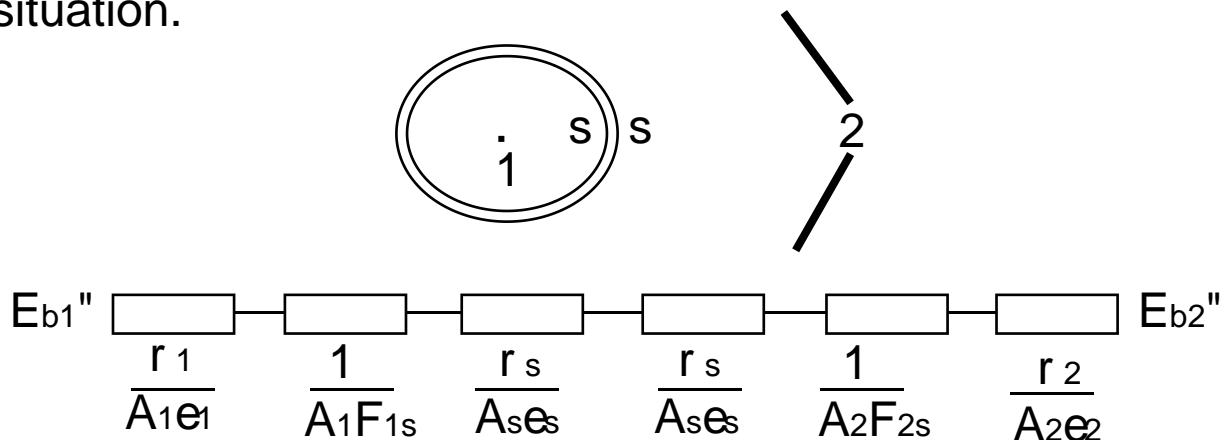
(3) Radiation shields

Often radiation needs to be minimised.

For example, a thermocouple may be required to measure only gas temperature in a radiant environment.. It may emit or receive radiation from its surroundings and give false readings.

In such a case a thin shield of low emissivity may be placed between the two bodies.

This gives rise to a single flow path electrical analogy situation.



T_s may be found by equating energy flows to and from the shield.

(4) Gas Radiation

In all of our work it has been assumed that $t=1$ for intervening media.

For many situations, and in particular for combustion problems involving CO_2 , CO , H_2O and SO_2 radiation is very important and should be taken into account.