

A note on the probability distribution function of the surface electromyogram signal[☆]

Kianoush Nazarpour^{a,1,*}, Ali H. Al-Timemy^{b,1}, Guido Bugmann^b,
Andrew Jackson^a

^a*Institute of Neuroscience, Newcastle University, United Kingdom*

^b*Centre for Robotics and Neural Systems (CRNS), Plymouth University, United Kingdom*

Abstract

The probability density function (PDF) of the surface electromyogram (EMG) signals has been modelled with Gaussian and Laplacian distribution functions. However, a general consensus upon the PDF of the EMG signals is yet to be reached, because not only are there several biological factors that can influence this distribution function, but also different analysis techniques can lead to contradicting results. Here, we recorded the EMG signal at different isometric muscle contraction levels and characterised the probability distribution of the surface EMG signal with two statistical measures: bicoherence and kurtosis. Bicoherence analysis did not help to infer the PDF of measured EMG signals. In contrast, with kurtosis analysis we demonstrated that the EMG PDF at isometric, non-fatiguing, low contraction levels is super-Gaussian. Moreover, kurtosis analysis showed that as the contraction

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*Corresponding Author

Email address: k.nazarpour@ncl.ac.uk (Kianoush Nazarpour)

¹Equal Contribution

force increases the surface EMG PDF tends to a Gaussian distribution.

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1 **1. Introduction**

2 A surface electromyogram (sEMG) signal is the electrical manifestation
3 of the neuromuscular activity and is recorded non-invasively from the surface
4 of the skin (Hogan and Mann, 1980; deLuca, 1979). The sEMG signal has
5 been extensively used for estimation and interpretation of the neural drive to
6 muscles (Merletti et al., 1999), extraction of a voluntary command signal for
7 control of prosthetic devices for individuals suffering from limb amputation
8 (Heftner and Jaros, 1988; Park and Meek, 1995; Huang et al., 2005), and in
9 biofeedback experiments in which the subjects learn to change patterns of
10 voluntary muscle contraction (Ince et al., 1984; Radhakrishnan et al., 2008;
11 Bloom et al., 2010; Nazarpour et al., 2012).

12 Conventionally in the prosthetic control applications after a pre-processing
13 stage, several features are extracted from the EMGs and a decoder is trained
14 to recognize different patterns of muscle activity. Various features in time
15 and frequency domains have been introduced for this purpose - for a review
16 see Micera et al. (2010). Higher order statistics (HOS) (Mendel, 1991) of
17 the EMGs have also proved effective in movement classification (Nazarpour
18 et al., 2005b, 2007). The merit of such HOS-based approaches lies in their
19 capability of capturing the skewness and peakedness (and other higher or-
20 der statistics) details of the EMG PDF that are ignored when the EMG is
21 assumed to be Gaussian process and consequently the first- and the second-

1 order moments and cumulants (i.e., mean, correlation, and variance) and
2 their spectral representations are analysed only.

3 Despite the success of HOS-based methods, there is not yet a general
4 consensus upon the PDF of the EMG signals to justify the application of
5 these statistics. For instance in (Roesler, 1974), it was shown that a Gaussian
6 density function can precisely model the EMG PDF at various contraction
7 strengths. Parker et al. (1977) also showed that EMG recorded at reasonably
8 low contraction levels can be modelled with a Gaussian process. In contrast
9 Hunter et al. (1987) and Bilodeau et al. (1997) used kurtosis analysis and
10 reported that during low intensity isometric contractions the PDF of the
11 sEMG signal is more peaked near zero than a Gaussian distribution. They
12 also reported that there was tendency for the kurtosis values to decrease with
13 increasing contraction level implying that the EMG PDF becomes closer to a
14 Gaussian distribution since the third- and the fourth-order statistics of a pure
15 Gaussian process are equal to zero. Clancy and Hogan (1999) also showed
16 that the PDF of the EMGs recorded during constant-angle, constant-force,
17 and non-fatiguing contractions falls between the Gaussian and the Laplacian
18 densities. Negentropy analysis of the EMG signals (Nazarpour et al., 2005a;
19 Naik et al., 2011) showed that the non-Gaussianity level of the EMG signal
20 depends on the muscular contraction level such that the increment in the
21 contraction level shifts the EMG PDFs towards the Gaussian distribution.

22 Kaplanis et al. (2000) explored the EMG PDF by investigating the bi-
23 coherence index of the EMG measurement. However, they arrived at the
24 conflicting result that the EMG signal is more non-Gaussian at low and high
25 levels of force while being in its maximum Gaussianity at the mid-level (50%)

1 of maximum voluntary contraction (MVC). Recently in Hussain et al. (2009),
2 the bicoherence analysis was used to test the Gaussianity of the EMG sig-
3 nals and it was shown that the EMG becomes less Gaussian with increased
4 walking speed force (increase in mean voluntary contraction).

5 In this paper, we revisited this problem and investigated the suitabil-
6 ity of the bicoherence of the sEMG signal for characterization of the non-
7 Gaussianity level of the sEMG signals for different levels of muscular activity.

8 **2. Method**

9 *2.1. Participants*

10 Four right-handed subjects (two female; mean age: 26 ± 5 years) partic-
11 ipated in the study. They were free of any history of neurological or motor
12 disorders and gave informed consent. The study was approved by the local
13 ethics committee at the Institute of Neuroscience, Newcastle University.

14 *2.2. Experimental setup*

15 Subjects controlled a myoelectric cursor (Radhakrishnan et al., 2008;
16 Nazarpour et al., 2012) by making isometric contractions of a single right
17 upper-limb muscle. We recorded surface EMG signals (Bio-logic disposable
18 snap electrodes, Natus Medical Inc.) from Abductor Pollicis Brevis (APB:
19 abducts the thumb) and Flexor Carpi Radialis (FCR: flexes the wrist) mus-
20 cles. Subjects completed two independent runs of the experiment (6 blocks),
21 one for each muscle as the controlling effector. The skin was cleansed with
22 alcohol beforehand and the electrode locations were chosen to maximize the
23 quality of recording. EMG measurements were amplified (gain 1K - 10K)

1 and high-pass filtered at 30 Hz (Neurolog NL824, Digitimer) before sampling
2 at 10 KHz (PCI-6071E, National Instruments). The hand was restrained in
3 an open, pronated posture inside a glove fixed to a horizontal board and the
4 forearm was strapped to the arm-rest of the chair. At the start of the exper-
5 iment, subjects were informed of the general structure of the experiment.

6 In the first (of six) block we asked the subjects to produce five contractions
7 with their maximum voluntary contraction level (MVC) for a period of two
8 seconds (100% MVC). In the second block, we instructed the subjects to
9 contract the muscle at a slightly lower level than in the first block. As
10 will be mentioned later in the results section, subjects on average produced
11 an activity of only about 50% MVC. They repeated the same procedure in
12 the fifth and the sixth blocks. In these four blocks no visual feedback was
13 provided.

14 At the start of the third block, subjects were instructed to produce com-
15 fortable levels of contraction of each muscle which they would be able to
16 repeat many times without fatigue. This corresponded to approximately 5%
17 to 10% of their maximum voluntary contraction level of that muscle. The
18 true contraction levels were verified offline. In the third and fourth blocks
19 (each of 100 trials), the subjects controlled the position of a myoelectric cur-
20 sor along a 1D vertical task space. The control signal was computed every
21 13ms by smoothing (with a rectangular window) the preceding 500ms of
22 rectified EMG. Subjects initiated a trial by relaxing the controlling muscle
23 to bring the cursor to a starting zone and remaining there for 250ms after
24 which a target appeared. The remainder of the trial was divided into two
25 fixed periods of one and three seconds, designated movement and hold peri-

1 ods. Auditory tones cued the start of the movement and hold periods. At
 2 the end of each trial, subjects received a score reflecting the proportion of the
 3 hold period that the cursor was inside the target and were instructed to max-
 4 imize this score. In each trial, a target was presented in one of five possible
 5 positions along the vertical axis; the order of the targets was pseudo-random.
 6 Targets one to five could be reached by producing an activity (with thumb
 7 abduction or wrist flexion whichever instructed) as large as one to five times
 8 comfortable contraction level, respectively. In approximately 2% of trials,
 9 subject could not hold the cursor inside the target area. We excluded these
 10 trials from analysis. Visual feedback was available throughout block 3 and
 11 4.

12 *2.3. Offline verification of contraction levels*

13 In contrast to earlier studies in which the EMG signals were recorded
 14 at fixed contraction level e.g. 25%, 50% MVC, we allowed the subjects to
 15 determine their comfortable contraction level required to hold the cursor in
 16 target 1. These comfortable contraction levels were different across subjects
 17 and muscles. We determined the actual contraction percentage by calculating
 18 the average mean absolute value (MAV) of EMG during the hold period for
 19 each target (20 presentations). After adjusting for the amplifier gain, we
 20 normalized these MAVs to the MVC activity (averaged over the 5 trials) (in
 21 each subject and for each muscle) with

$$\% \text{ of } MVC = \frac{\frac{1}{20} \sum_{i=1}^{20} MAV_i}{\frac{1}{5} \sum_{j=1}^5 MVA_j \text{ of } 100\% \text{ MVC}} \quad (1)$$

1 *2.4. Bicoherence Analysis*

2 A frequency-domain measure of the third-order cumulant $C_3^{\mathbf{x}}(m, n)$ is the
 3 bispectrum (Hinich, 1982) and is calculated by taking a two-dimensional
 4 discrete-time Fourier transform from $C_3^{\mathbf{x}}(m, n)$ with

$$B^{\mathbf{x}}(w_1, w_2) = \sum_{m, n=-\infty}^{+\infty} C_3^{\mathbf{x}}(m, n) e^{-j(w_1 m, w_2 n)}. \quad (2)$$

5 The normalized bispectrum is called bicoherence and is computed with

$$Bic^{\mathbf{x}}(w_1, w_2) = \frac{B^{\mathbf{x}}(w_1, w_2)}{P^{\mathbf{x}}(w_1)P^{\mathbf{x}}(w_2)P^{\mathbf{x}}(w_1 + w_2)} \quad (3)$$

6 where $P^{\mathbf{x}}(w)$ denotes the power spectrum of \mathbf{x} at frequency w . Bicoherence
 7 can be used to measure the skewness of a random process (Mendel, 1991).
 8 For that purpose, a test of Gaussianity was defined in (Hinich, 1982) by the
 9 mean bicoherence power

$$S^{\mathbf{x}} = \sum_{w_1, w_2} |Bic^{\mathbf{x}}(w_1, w_2)|^2 \quad (4)$$

10 and is compared with a central chi-squared distribution; in essence if $Bic^{\mathbf{x}}(w_1, w_2)$
 11 is zero then the $S^{\mathbf{x}}$ statistic is a central chi-squared distributed random vari-
 12 able with two degrees of freedom - see (Hinich, 1982) for mathematical proof.

13 *2.5. Kurtosis Analysis*

14 The kurtosis of a random variable is computed by dividing its fourth cu-
 15 mulant by the square of its second cumulant. Sample kurtosis for a univariate
 16 random process “ \mathbf{x} ” can be estimated with

$$kurt_{\mathbf{x}} = \frac{E\{\mathbf{x}^4\}}{E\{\mathbf{x}^2\}^2} - 3 \quad (5)$$

1 where $E\{\cdot\}$ denotes the statistical expectation operator. Kurtosis measures
2 the peakedness of a PDF.

3 A MATLAB R14-based graphical user interface linked to Cogent (2000)
4 was developed to control this experiment. All data analysis was carried out
5 in MATLAB.

6 **3. Results**

7 Figure 1A shows a representative set of raw EMG recorded from APB in
8 one subject for different contraction levels. Figure 1B depicts the probability
9 distribution functions that are estimated using the kernel smoothing method
10 (Parzen, 1994) with Gaussian kernels. For comparison purposes, the PDF
11 of a random variable of the same length drawn from a normal distribution
12 is also depicted. Note that in Figure 1, only for clarity of presentation, all
13 signals are standardized to zero mean and unit variance. This operation has
14 no effect on the higher order statistics of these signals but renders the vertical
15 axes in Figure 1 (A and B) arbitrary.

16 Figure 2 (A, B) displays the computed mean of kurtosis values of the APB
17 and FCR muscle activity relative to the percentage of the MVC activity
18 for individual subjects. Importantly, the mean of kurtosis reduced for all
19 subjects and in both muscles when the contraction level increased reflecting
20 a shift from a non-Gaussian distribution to a more Gaussian-like distribution.
21 A two-way (muscle and contraction level) ANOVA test confirmed the main
22 effect of contraction level (repeated measures, $F_{6,18} = 87.37$, $p < 0.001$, $n =$
23 4). The main effect of muscle was not significant ($F_{1,3} = 0.927$, $p = 0.40$, $n =$
24 4). Figure 2 (C,D) show the mean bicoherence indices computed for APB

1 and FCR muscles for different force levels. In contrast to (Kaplanis et al.,
2 2000; Hussain et al., 2009), we did not observe any consistent trend in mean
3 bicoherence index relative to contraction level ($F_{6,18} = 2.51$, $p > 0.05$, $n = 4$)
4 in either muscle.

5 **4. Concluding Remarks**

6 By analysis of the kurtosis of the EMG signals we showed that at low con-
7 traction levels, EMG PDFs are more peaked at zero. When the force level
8 increases, the EMG PDF tends to a more bell-shaped Gaussian distribution.
9 Related physiological work have shown that increasing the force level will
10 not only increase the rate of the already firing motor units (temporal recruit-
11 ment), but also recruits more motor units of same or other types (Fuglevand
12 et al., 1993). The central limit theorem (CLT) predicts if sufficiently large
13 number of (independent) motor units fire, the signal recorded from the sur-
14 face of the skin will be approximately normally distributed. Our results are
15 consistent with the predictions of the CLT.

16 Several earlier studies show that the sEMG signal irrespective of the con-
17 traction force level exhibits a symmetric distribution function that leads to
18 small skewness $C_3^x(m, n)$ values (See Nazarpour et al., 2007 and reference
19 therein). Authors of (Kaplanis et al., 2000) and (Hussain et al., 2009) over-
20 looked the fact that the so-called bispectrum index-based Gaussianity test
21 (Hinich, 1982) only quantifies the skewness of a probability. Therefore, the
22 Gaussianity test in (Hinich, 1982) may only be used to reject the Gaussian-
23 ity null hypothesis. If the bispectrum index is zero, the Gaussianity of the
24 process may not be inferred since fourth and higher-order cumulants and

1 polyspectra would not necessarily be zero (Mendel, 1991). For instance, if a
2 signal has a Laplacian distribution, the bispectrum and all the odd-ordered
3 polyspectra are zero, however, the even-ordered statistics (e.g. kurtosis) or
4 polyspectra (e.g. the trispectrum) will not identically be equal to zero.

5 In contrast to (Bilodeau et al., 1997; Clancy and Hogan, 1999; Nazarpour
6 et al., 2007) in which the EMG signals were recorded at fixed percentages of
7 the maximum contraction level (MVC), we deliberately recorded the EMGs
8 in a more flexible range of the force levels so that we can quantify the PDF
9 of the sEMG signals in a broader range of force levels.

10 We characterized the PDF of the EMG signals at different contraction
11 levels in two muscles. However, the choice of the muscle should not influ-
12 ence our main results significantly. Sanger (2007) and also we in Nazarpour
13 et al. (2007) examined the PDF of Biceps and Triceps muscles at different
14 contraction levels and arrived at a comparable result that a Laplacian distri-
15 bution is more suitable for EMG PDF modelling measured at low contraction
16 levels. However, not only other biomechanical factors, such as contraction
17 speed and isometricity of contraction, but also several anatomical, e.g., num-
18 ber of active motor units, size of the motor units, the spatial distribution of
19 motor units and physiological factors (neural disorder and fatigue) can influ-
20 ence the shape of the EMG PDF. In addition, the measurement noise (e.g.
21 crosstalk and electronic interferences) can change the PDF of the recorded
22 signals. These factors might explain the lack of consensus upon the EMG
23 PDF in literature.

24 The demonstration that the PDF of the sEMG signal recorded at low
25 forces is closer to a Laplacian distribution may have significance for prosthe-

1 sis control or biofeedback experiments since this could form a flexible sub-
2 strate for developing novel mathematical tools tailored for super-Gaussian
3 processes such as higher order statistics. For instance, Sanger (2007) de-
4 veloped a Bayesian algorithm to predict the envelope of the EMG signals
5 and showed that by assuming an exponential density (half-Laplacian) for the
6 sEMG signal the output of a Bayesian filter follows the rapid changes in the
7 EMG amplitude much faster than the conventional linear approaches. Nev-
8 ertheless, successful use the HOS of surface EMG for prosthesis for control
9 and biofeedback depends on the reliability of the algorithms that estimate
10 these statistics accurately. Our current work includes developing robust and
11 efficient algorithms to estimate recursively the sample kurtosis value in real-
12 time.

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Figure 1
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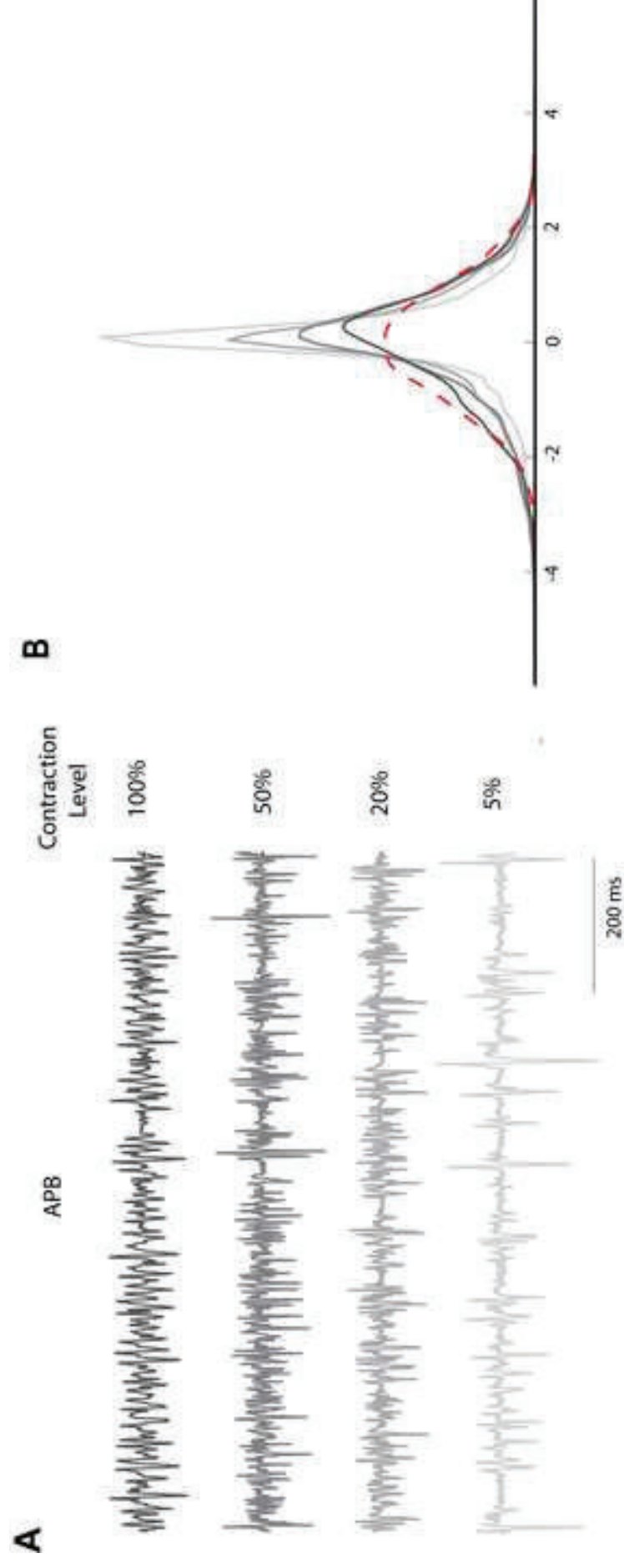


Figure 2
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