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Memorizing and recalling spatial–temporal patterns in an oscillator model of the hippocampus

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Abstract

We describe the model of the hippocampus consisting of interactive oscillators with input from the entorhinal cortex (modulating the main information flow by a theta rhythm) and the septum (a theta rhythm generator). When interconnections between oscillators are allowed to strengthen in an adaptive way, the network can be trained using a series of lessons. This results in a connection matrix that memorizes the temporal sequence of inputs. Presenting one of the lessons to the trained network results in reproduction of the remainder of the sequence. In this paper, we create such a connection matrix, derive from it an appropriate Markov chain and simulate the chain to illustrate its dynamics. © 1998 Elsevier Science Ireland Ltd. All rights reserved.

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1. Introduction

We study memorization and recall based on an oscillatory model of the hippocampus. We take the natural frequency of each oscillator to be 40 Hz (gamma rhythm), the septal input is oscillatory with frequency ≈ 5 Hz (theta rhythm), and the input from the entorhinal cortex deliveries information for memorizing and recall. These assumptions are in good correspondence with experimental data (Traub and Miles, 1991; Buzsa'ki et al., 1994; Bragin et al., 1995; Vinogradova, 1995).

We present a series of lessons to the oscillatory model and allow connections between oscillators to strengthen or weaken according to whether there is synchrony between the two oscillators or not. Each lesson L_j corresponds to a particular pattern of theta rhythm firing. We present a first lesson to the array. Next, we present a second lesson, coincident with the first, for an interval of time that is comparable to one theta cycle. Then only the second lesson alone, etc. A sequence of lessons $L_1 \rightarrow L_2 \rightarrow \ldots \rightarrow L_N$ results in a connection matrix that has a left-to-right structure; it cap-

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tures both the individual lessons and the transitions between them and it records a memory of the temporal sequence. Details of this work is presented elsewhere (Borisyuk and Hoppensteadt, 1998).

Here we derive a Markov chain, based on this connection strength matrix, by introducing a transition matrix having the same left-to-right structure as the connection matrix. Its sample paths describe the memorized lessons in sequence, but in the presence of random noise.

2. Oscillatory model of the hippocampus: global description

Neurophysiological and neuroanatomical evidence shows the hippocampus to have a complex three-dimensional structure (Isaacson, 1982; Amaral and Witter, 1989). We visualize the hippocampal formation as being a three-dimensional brain structure with a long septotemporal axis and two axes, which are perpendicular to the long axis. Each two-dimensional slice perpendicular to the long axis intersects three main regions called the dentate gyrus (DG), the CA_1 field, and the CA_3 field. These regions include both excitatory pyramidal neurons and inhibitory interneurons. The connections between hippocampal neurons are as extensive and highly organized in the septotemporal axis of the hippocampus as in the transverse axes (Amaral and Witter, 1989). The neural populations located in each region can support endogenous oscillatory behavior without any external input and some experimental evidence suggests that their natural frequency is in the range of gamma rhythms (40–70 Hz) (Traub and Miles, 1991; Buzsa'ki et al., 1994).

In this paper, we neglect the fine structure of the two-dimensional slice transverse to the long axis. Instead, we model the interactive excitatory and inhibitory neural populations in a thin slice as being an oscillator and we model the hippocampus as being a chain of coupled oscillators S_1, S_2,..., S_N distributed along the septotemporal axis.

Two primary inputs to the oscillator are considered here: input from afferents of the entorhinal cortex, and input from the medial septum. The input from entorhinal cortex delivers highly preprocessed information about sensory stimuli and both supply low frequency (3–9 Hz) theta rhythm activity (Vinogradova, 1995; Iijima et al., 1996).

The influence of low frequency oscillatory inputs results in complex dynamics of the model including entrainment, chaotic behavior and envelop oscillations.

Phase deviations arise between external input signals when they are distributed along the hippocampus. The result is that the net input to a oscillator involves a signal having a theta rhythm frequency and a phase deviation that is related to its location along the septotemporal axis (O'Keefe and Recce, 1993; Bragin et al., 1995).

From a mathematical point of view, this model of the hippocampus is described by a system of interacting oscillators having two external oscillatory inputs. Due to the time-delays in the input signal propagation, two travelling waves arise and propagate along afferents as inputs to the hippocampus but in opposite directions. The interaction of these waves makes possible synchronization of oscillators, coincidence and non-linear resonance. In particular, the activity level of some oscillators increases due to specific relations between frequencies of inputs and phase deviations between them. In this sense, the hippocampal model works as a spatial comparator of two input signals and in turn reacts with theta rhythm synchrony or not (Fig. 1).

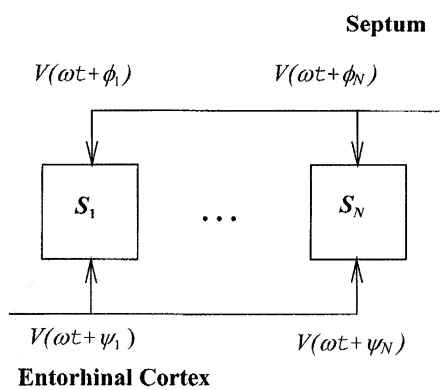

Fig. 1. An oscillatory model comprising N identical oscillators that have inputs from the septum and from the entorhinal cortex, that have a fixed wave form (V), a fixed frequency (ω) and a phase deviation ϕ_j or ψ_j. The phase differences along the line are $\phi_j - \psi_j$ for $j = 1,..., N$.

3. Training and recall of temporal sequences

We consider this model when the oscillators are linked by connections (described by a matrix **C**) that can strengthen or weaken in response to the presence or absence of synchronous activity. In a previous study (Borisyuk and Hoppensteadt, 1998), we show that when a temporal sequence of lessons is presented to such a network, then it changes dynamically and stores the lessons in a special form within the connection strengths. We demonstrate here how this occurs, the results for the connection matrix and how this system can recall the temporal sequence once it is stimulated with input comparable to one of the lessons. This is similar to short term memory, since, in our model, the connection strengths can weaken in the absence of synchronous activity.

The connection matrix that emerges from training has an interesting structure (possibly after re-labelling of rows and columns). Elements in the diagonal blocks describe the dynamics required by a lesson and elements in super-diagonal blocks describe the transition from one lesson to the next in the temporal sequence. Thus, the connection matrix stores both the lessons and the sequence in which they appear in a left-to-right structure that is familiar from communications applications (Nadas and Mercer, 1996).

We suppose that the brain stores this connection structure through some mechanisms unknown to us. We abstract this structure in the next section by creating a Markov chain whose transition probability matrix is based on the emergent connection matrix, but first we illustrate the training process and the formation of **C** using a phase oscillator model.

3.1. Training

Let L denote a lesson. The lesson should be a set phase deviation between the sources of oscillation at the cortex and septum. This would give a pattern of theta rhythm activity. For our purposes here, we describe it as a patterned vector of inputs, namely a vector of 1's and 0's, with 1 indicating which oscillator is being stimulated. We suppose that there are M lessons, say L_1, L_2,..., L_M. Each lesson will result in a particular pattern of theta rhythm activity among the oscillators. There are two parts to learning a temporal sequence. There are the individual messages to be learned, L_j and there are the transitions between lessons to be learned, say $L_1 \rightarrow L_2$, $L_2 \rightarrow L_3$, $L_3 \rightarrow L_4$, etc. Training includes both kinds of these events.

Thus, we present the first lesson for a period of time sufficient for the connection matrix components to come near equilibrium, and then we present both the first and second lesson for period of time, then the second alone, etc. These time windows have length ≈ 200 ms, the period of a theta rhythm.

3.2. Recall

The process of recalling a memorized sequence begins with application of the first learned pattern through cortical input and it is held constant during one time window. Due to the connection weights, this pattern activates the second pattern, which appears with a small delay, the second pattern activates the third pattern, etc. Thus, the pattern initially presented activates sequentially all of the following patterns in the sequence in the correct order with small delays. This means that during the recall process only lessons following the one stimulating the network will appear, but no previous patterns.

3.3. Example

We will illustrate our results using here specific results for a frequency domain model (Hoppensteadt, 1997), but comparable results using several other models are obtained by Borisyuk and Hoppensteadt (1998). These mechanisms result in creation of a connection matrix **C** for models based on frequency domain methodology. We illustrate training here for a network of voltage-controlled oscillator neuron models (VCONs).

The system governing this model is

$$\frac{\mathrm{d}x_j}{\mathrm{d}t} = \gamma[1 + aL^j_m(t)\cos(x_j - \omega t)] + \sum_{i=1,\, i\neq j}^{64} C_{ij}\cos x_j \cos x_i$$

$$\frac{dC_{ij}}{dt} = b \cos x_i \cos x_j$$

here x_j is a phase of jth oscillator; γ is a natural frequency of the oscillator ($\gamma = 40$); ω is frequency of the septal input ($\omega = 5$); $L^j_m(t)$ is cortical input signal to the jth oscillator by the mth lesson at moment t ($L^j_m(t)$ is binary); a and b are constant coefficients ($a = 1.1$, $b = 0.01$); C_{ij} is connection strength from oscillator i to the oscillator j; i, $j = 1,2, \ldots, N$.

The sequence $L(\mathrm{t}) = (L_1, L_2, \ldots, L_M)$ presents to the network a temporal sequence of lessons. In this simulation, L presents the following sequence: $\mathrm{L}_1 = \mathbf{1}(1{:}15) \rightarrow \mathrm{L}_2 = \mathbf{1}(11{:}26) \rightarrow \mathrm{L}_3 = \mathbf{1}(49{:}64)$, where $\mathbf{1}(11{:}26)$ indicates a vector of 64 elements, of which those with indices from 11 to 26 are 1 and the rest are 0, etc. We suppose here that decay of the connections once formed is slow compared to the processes modelled here, so this term is ignored in the simulation. The emergent connection matrix is described in Fig. 2 (b). It records these lessons and the transitions between them. In this simulation, each of the three lessons, including the overlap transition lessons, is presented for 20% of the training session. Thus, the two overlaps of lessons are also presented for 20% of the session. A training algorithm similar to the one used here was introduced and used for a different model (Amit, 1989).

The connection matrix generated by this system is shown in Fig. 2. In this case, overlapping lessons are presented to the network, and the result is a left-to-right structure. In this case, overlapping lessons (L_1 and L_2) are presented; they result in very strong connections emerging at the overlap oscillators.

4. Long term memory

Without specifying the process by which short-term memory is converted into long term memory, we abstract this network by identifying from it a Markov chain. In particular, we begin with the connection matrix as depicted in Fig. 2, replace each diagonal block with a recurrent chain with leakage and replace each super-diagonal block with a transient chain. The result is a stochastic matrix. We might view this as being embedded in the cortex as a result of reinforcement of the short-term memories described above, during some other brain processes.

In the general case, the resulting matrix, say $\mathbf{P}$, has the form (possibly after reordering the labels):

$$\begin{pmatrix} P_{1,1} & \mu B_{1,2} & 0 & 0 & 0 & 0 \\ 0 & P_{2,2} & \mu B_{2,3} & 0 & 0 & 0 \\ 0 & 0 & P_{3,3} & \mu B_{3,4} & 0 & 0 \\ \vdots & \cdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & P_{M,M} \end{pmatrix}$$

where the row sums are all equal to 1, the components are all non-negative, and the parameter μ, which indicates the probability of transition from one state to another, reflects the duration of training for transition from one lesson to the next. In the simulation above, $\mu = 0.005$. The matrix describes the first lesson learned—it might result in the brain governing some particular search program. The matrix $\mu B_{1,2}$ describes the probability of transition from lesson L_1 to lesson L_2, etc.

This Markov chain will describe passage through the memorized states, but in the presence of random noise. So, the chain is related to the lessons learned by the model with the addition of random noise that reflects the possibility of continuing to perform one learned task repeatedly, or to move to the next task, etc.

This left-to-right structure of the chain is familiar in the engineering literature (Nadas and Mercer, 1996). We can also consider this chain in the presence of further random noise perturbations. Specifically, we consider the new probability transition matrix

$$\mathrm{P}^*(n, \varepsilon) = \mathrm{P} + \varepsilon R_n$$

where ε is a small positive parameter. We suppose that the sequence $\{R_n, n = 1,2, \ldots\}$ is an ergodic, stationary matrix-valued random process. It is shown by Hoppensteadt et al. (1998) that for such a system, there is a matrix $\mathbf{\Pi}$ such that

$$\lim_{\varepsilon \to 0} \limsup_{n \to \infty} \prod_{k=1}^{n} \mathbf{P}^*(k, \varepsilon) = \mathbf{\Pi}$$

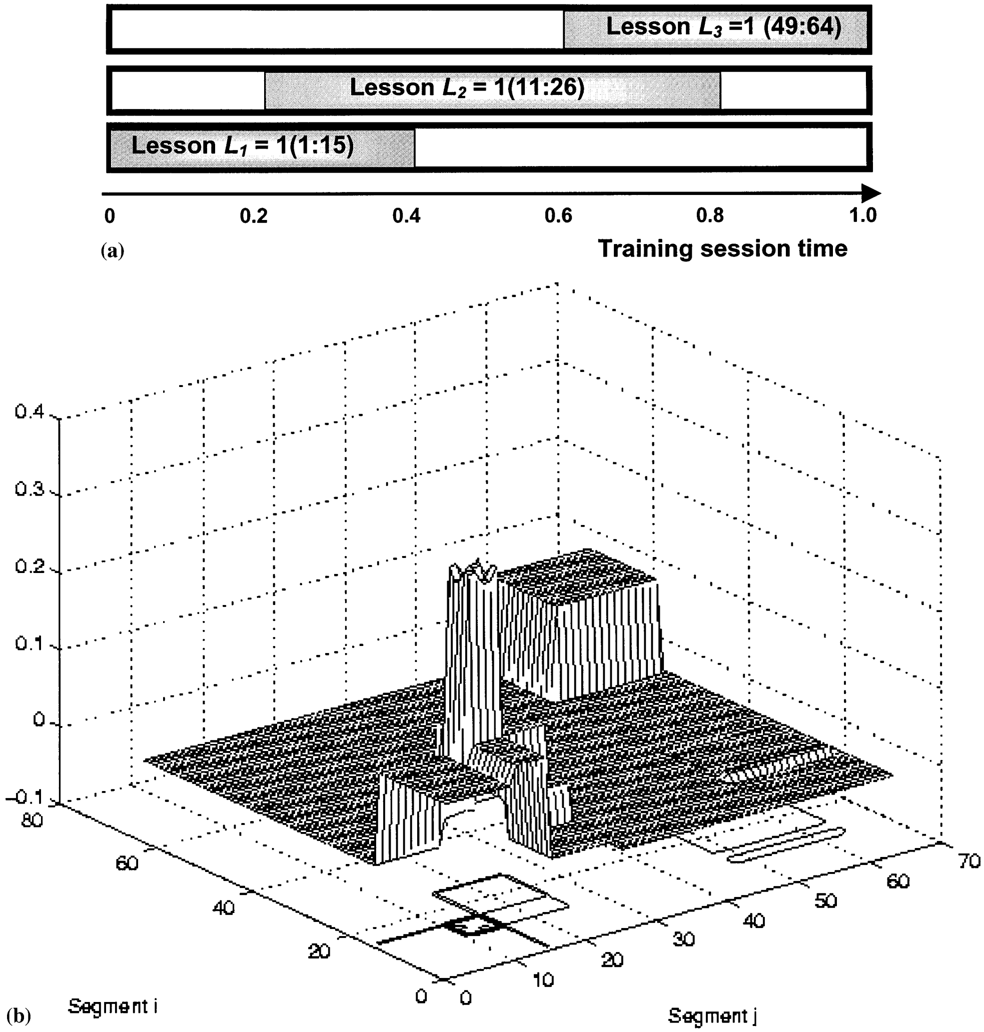

Fig. 2. (a) The example of spatio–temporal pattern of a training sequence consisting of three lessons with 20% time overlap is shown. (b) Graph of the connection matrix **C** generated by VCON model with overlapping lessons. The contour mapping of this matrix is shown below the graph. The segments are labeled 1:64. The height of the surface over index (j, i) is $C_{j,i}$. The first raised section near (0, 0) describes the connections strengthened by lesson 1. The off-diagonal block from $j = 1{:}15$ and $i = 11{:}26$ describes the joint learning of lessons 1 and 2, etc.

with probability one. If the structure of **P*** is, to leading order, purely left-to-right, then **Π** corresponds to a Markov chain having an absorbing block. The structure of **Π** can be determined once the structure of the absorbing blocks is known. The probability distributions among states change in time. Their initial behavior is described by **P**, their intermediate time behavior is described by a diffusion process derived from the central limit theorem (not discussed further here, see Hoppensteadt et al. (1998), and their long time behavior is described by **Π**. Thus, the sample paths generated using **Π** approximate the eventual behavior of sample paths of all of the chains for sufficiently small values of ε.

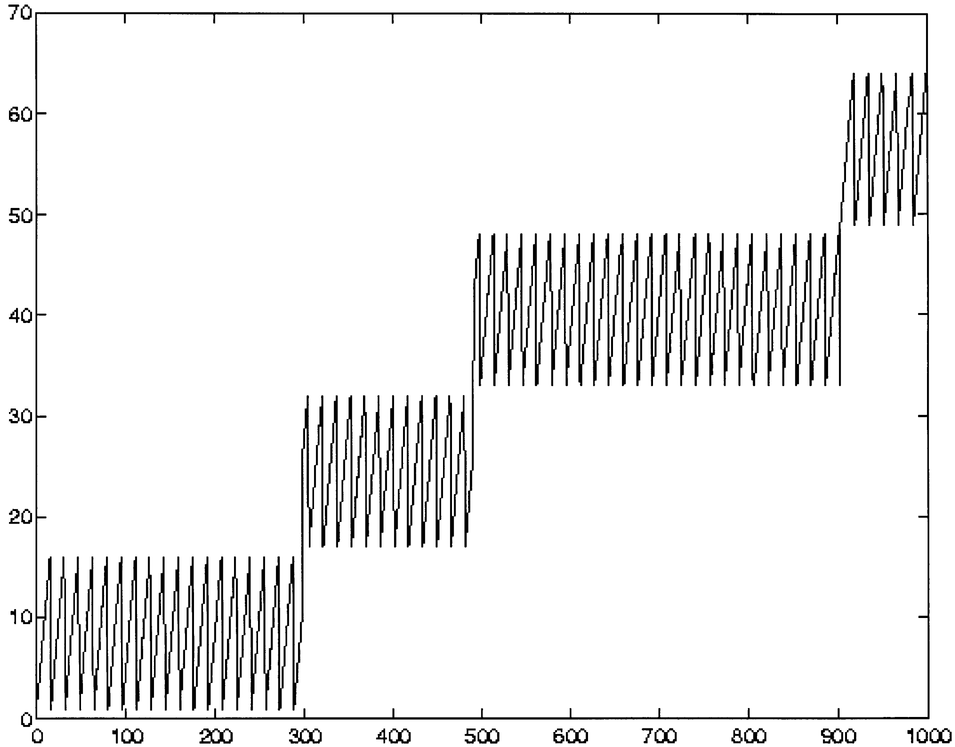

Fig. 3. Sample path generated by the chain.

4.1. Example

We use this Markov chain approach to simulate a navigation experiment using behaving rats, by generating an analog of place cell dynamics (O'Keefe and Nadel, 1978; McNaughton et al., 1996). Consider the case of four blocks each having 16 steps ($N = 64$), and suppose that activity in each block corresponds to search activity of the animal in a certain section of a square room. We first generate the transition probability matrix, memorizing the information related to the location in the room and then a sample path of it, and then we map the sample path to a search trajectory in the room. This latter mapping, which we refer to as being the place cell mapping, is subjected to small amplitude noise in our simulations [Gaussian noise with variance comparable to $O(\varepsilon^2)$].

The results of a sample calculation are shown in Fig. 3, where a sample path of the process is shown, and Fig. 4, where the resulting spatial search pattern is depicted. In the first, a sample path of the process generated by a Markov chain where **P** has four recurrent blocks, each of 16 steps. Therefore, the size of **P** is 64×64. The second plot shows a search algorithm for a square room that is generated by this chain, but perturbed by small random noise. Each of the blocks in **P** corresponds to searching in one quadrant of the room, the steps in the chain are each a search movement, and small noise perturbs the searching path, sometimes leading to moving to another block.

In this case, the matrix **Π** has the form of all rows being identical with zeros in the first 48 columns and the value 1/16 in each of the other 16 columns, reflecting that the last block is absorbing for this chain.

The use of Markov chains to model memory has a long history, e.g. see Anderson and Rosenfeld (1988) and Chetaev (1984). However, our use

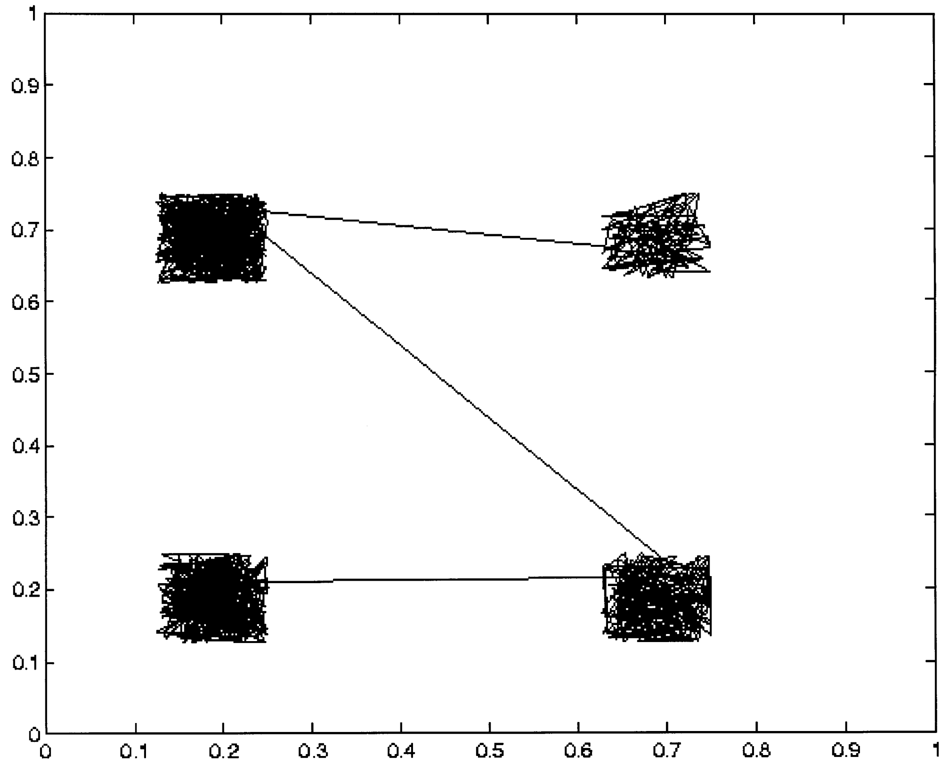

Fig. 4. Search path.

here is motivated by the structure emerging from a learning algorithm in a continuous time model. Moreover, the recent results of Hoppensteadt et al. (1998) enable us to approximate the behavior of the chain in a useful way.

5. Discussion

The work presented here is based on an oscillatory model of the hippocampus. We present here a particular version of it using canonical frequency domain variables. We have, in a previous study (Borisyuk and Hoppensteadt, 1998), obtained similar results for training using several other models of oscillators based on integrate-and-fire, Wilson–Cowan, and other canonical models (Hoppensteadt and Izhikevich, 1997) of neural networks. All of these models exhibit patterns of phase locking of oscillator activity to inputs that reflect their location along the array due to the phase deviations between inputs, and we expect similar results when the Hodgkin–Huxley or Traub–Miles models are used. Patterns of phase locked behavior are observed in experiments on rat hippocampus (Bragin et al., 1995), and phase shifts are known in the firing of place cells (O'Keefe and Recce, 1993). Those experiments guide us in the selection of parameters in our models for simulation.

The training algorithm described here accounts for lessons to be learned, as well as transitions between lessons in a temporal sequence. The emergent patterns of memorized states are reflected in the connection matrix that is formed between oscillators through training.

We have shown here how a Markov chain model can be constructed from the trained connection matrix and that the behavior of sample paths generated by the chain can be analyzed using approximation methods from the theory of randomly perturbed dynamical systems (Hoppensteadt et al., 1998). The example presented here is cast in terms of navigation, but there is no known connection between this example and data obtained from place cell experiments. On the other hand, it will be of interest to use Markov chains to describe observations in rat navigation in a room, and the methodologies of modelling and analysis described here might be useful for describing the dynamics of networks of place cells.

Because of space limitations here, we describe only training resulting from direct application of lesson vectors to the oscillators, not through the alteration of phase deviations between the signals generated in the septum and entorhinal cortex as we discuss in Borisyuk and Hoppensteadt (1998). Comparable results are obtained there when the lessons are applied to the network through phase deviations between these signals.

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