



## Oscillatory neural network model of attention focus formation and control

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### Abstract

A new mechanism to control attention focus formation and switching in the model of selective attention is suggested and studied. The model is based on an oscillatory neural network (ONN) with the star-like architecture and phase shifts in connections between oscillators. Attention is modelled as a dynamical mode of partial synchronisation between a particular subgroup of oscillators and the central oscillator (CO). A new theoretical method to study full and partial synchronisation in the system is presented. Equations for the frequency of synchronisation are derived which allow the programming of the dynamical behaviour of the system depending on the parameters. In particular, we show that phase shifts in connections between oscillators provide an efficient mechanism of attention control.

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**Keywords:** Oscillatory neural network; Synchronisation; Phase shifts; Selective attention

### 1. Introduction

Animals and humans display a wide spectrum of rhythmic activity patterns in many areas and structures of the brain (see, e.g. Basar, 1998). Several hypotheses have been put forward about the role of these oscillations in information processing. Discovery of the phenomenon of synchronous oscillations in the cerebral cortex (Eckhorn et al., 1988; Gray et al., 1989; Singer and Gray, 1995) is considered as supporting the idea that these oscillations may offer a mechanism for feature binding. Other investigations reveal that synchronous oscillations can be a correlate of stimulus

selection by attention (Steinmetz et al., 2000; Fries et al., 2001).

Oscillatory neural networks (ONNs) provide appropriate facilities to model synchronisation effects in the brain (Borisyuk et al., 2002). In particular, networks of phase oscillators have been found efficient when a qualitative mathematical description of synchronisation is needed. A phase oscillator is described by one variable, the oscillation phase, and the interaction between oscillators is realised via phase locking. This simplifies the analysis of conditions when the synchronisation of oscillators takes place (Kuramoto and Nishikawa, 1987; Daido, 1988, 1990; Strogatz and Mirollo, 1988).

The model presented in this paper is a part of the project on the development of an oscillatory model of cognition. The partial scheme of the model corresponding to a subsystem of the visual information pro-

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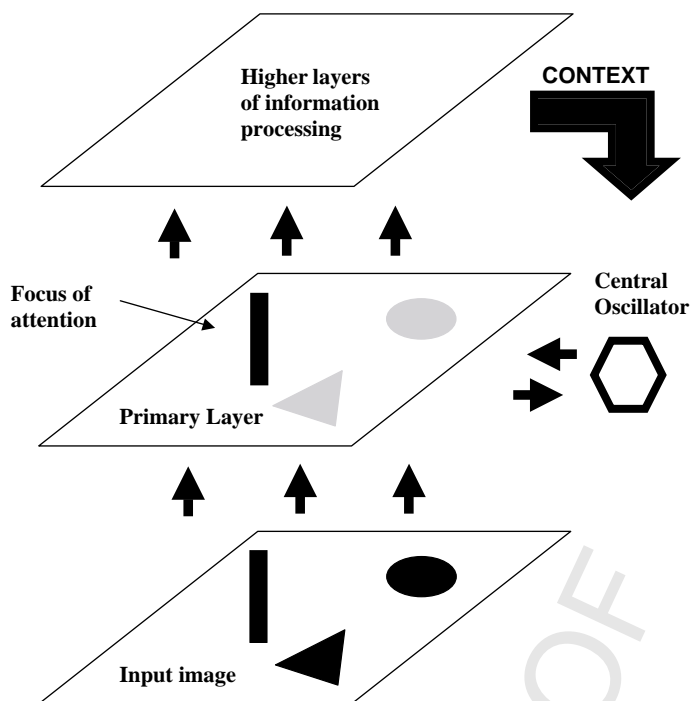


Fig. 1. The architecture of the network. The input image contains three objects. In the PL an object in the focus of attention is painted in black, other activated regions are painted in grey.

49 cessing is shown in Fig. 1. The Primary Layer (PL)  
 50 receives the information about the input image and  
 51 performs an early stage of information processing. At  
 52 this stage, the primary features of the objects com-  
 53 posing the image such as colour, brightness, contrast,  
 54 orientation, local shape, etc. are extracted. Also, the  
 55 attention focus is formed in the PL which means that  
 56 for some time one object is selected from the image  
 57 to be transmitted to the higher layers of processing  
 58 for further analysis (recognition, memorisation, nov-  
 59 elty detection, etc.).

60 Object selection is made on the basis of the fea-  
 61 tures and the additional information about the image  
 62 context. The context is important because it allows the  
 63 system to determine which features and which object  
 64 are most salient at the current moment. After the pro-  
 65 cessing of the attended object is over this object is ex-  
 66 cluded from consideration for a period of time giving  
 67 other objects the opportunity to be included in the at-  
 68 tention focus. For example, a controlling context sig-  
 69 nal such as “black vertical bar” biases the equilibrium  
 70 of the attention system in such a way that a black ver-

tical bar has the highest priority to be selected into the  
 attention focus. Note that the context might be formed  
 at the early stage of information processing, which  
 corresponds to the so-called “fast forming attention”  
 (30–50 ms) with the focus on simple objects. Another  
 possibility is that the clarification of the context re-  
 quires more serious processing and the decision is  
 made at higher levels on the basis of pattern recogni-  
 tion, information retrieval from the memory, novelty  
 detection, etc. For example, the context such as “small  
 aeroplane” which appears in association with the spe-  
 cific sound of a small aeroplane requires a longer pe-  
 riod of information processing (about 200 ms).

We consider in detail the dynamics of an oscillatory  
 neural network which allows the system to form and  
 effectively control the attention focus. We suppose that  
 the attention system is represented by an ONN with  
 a central oscillator (CO). The CO plays the role of  
 the central executive of the attention system as it is  
 suggested in Cowan (1988). The ONN with a CO has  
 a star-like architecture of connections where global  
 interaction between the so-called peripheral oscillators

(POs) (representing the elements of the ONN different from the CO) is implemented through forward and backward connections with the CO. This architecture has important advantages which have been discussed in our earlier papers (Kazanovich and Borisyuk, 1994, 1999). In these papers we considered a simple case of two objects represented by two groups of POs with the possibility to switch attention from one group to another varying the natural frequency of the CO or the strength of the interaction between the CO and POs. The regime of partial synchronisation of the CO with one of two groups was considered as representing the situation when the attention is focused on one object.

In biological terms, POs might represent cortical columns in the areas of the visual pathway (striate, extrastriate, and higher). We postulate that the focus of attention is represented by the set of POs that work synchronously with the CO (Kryukov, 1991; Kazanovich and Borisyuk, 1999). We associate the CO with the septo-hippocampal system whose final position in the pyramid of cortical convergent zones (Damasio, 1989) gives it a direct or indirect access to different cortical areas.

A saliency-based approach to attention modelling has been suggested by Koch and Ullman (1985) and developed in detail in the papers (Niebur and Koch, 1998; Itti and Koch, 2000). An important distinction from our model is that this approach relies on a traditional winner-take-all procedure and implements “essentially bottom-up strategies for the rapid selection of the most conspicuous parts of the visual field”. In our model a phase-locking mechanism is used to organize the competition between the assemblies of POs for the synchronisation with the CO. The top-down influence of the CO on the activity of POs plays a crucial role in attention focusing.

In this paper we consider a generalisation of the ONN with a CO by introducing phase shifts in the interaction between the oscillators. We show that the phase shift may be a controlling parameter that determines the type of a synchronous regime (and the relative attention focus) that appears in the ONN. We theoretically investigate the dynamics in the ONN with a CO and find the formulas to compute the frequency of synchronous oscillations in the regimes of full and partial synchronisation. It is shown that phase shifts provide a flexible and effective control of attention focus formation and switching between different sets of

oscillators especially under conditions of the adaptation of the natural frequency of the CO.

## 2. Model description

The model of attention focus formation and control consists of a CO and many POs. There are forward and backward connections between the CO and POs which are characterised by both the connection strength and phase shift. For simplicity we do not consider local connections between POs. The dynamics of the system are described by the equations:

$$\frac{d\theta_0}{dt} = \omega_0 + \frac{A}{n} \sum_{i=1}^n \sin(\theta_i - \theta_0 + \gamma), \quad (1)$$

$$\frac{d\theta_i}{dt} = \omega_i + B \sin(\theta_0 - \theta_i), \quad i = 1, \dots, n, \quad (2)$$

where  $\theta_k$  ( $k = 0, 1, \dots, n$ ) are oscillator phases,  $\omega_k$  are the natural frequencies of oscillators,  $A$  and  $B$  are coupling strengths ( $A > 0, B > 0$ ),  $\gamma$  is a phase shift, and  $d\theta_k/dt$  describe the current frequencies of oscillators. Eq. (1) describes dynamics of the phase  $\theta_0$  of the CO and Eq. (2) correspond to the phases  $\theta_k$  of the POs. Note that the case of phase shifts in both feedforward and backward connections between the CO and POs can be reduced to Eqs. (1) and (2) by a simple change of the variables.

We suppose that the focus of attention is formed by those POs which work synchronously with the CO. Three types of synchronous dynamics are of interest in relation to attention modelling:

- *Full synchronisation*: all POs work synchronously with the CO (that is with the same current frequency for all oscillators);
- *Partial synchronisation*: there are some POs which work nearly synchronously with the CO but others are out of synchronisation;
- *No synchronisation*: all oscillators have different current frequencies.

The choice of a regime depends on the parameter values. We think that information processing in the brain can be represented as a sequence of different regimes of partial synchronisation: some neural assembly that works synchronously with the central executive for a short time is changed by another synchro-

181 nised assembly corresponding to new parameter val-  
 182 ues, etc. This sequence appears in response to chang-  
 183 ing demands in the operation of the cognitive system.

184 **3. Theoretical study of dynamical regimes**

185 We have developed a new theoretical approach to  
 186 the analysis of the dynamics of system (1)–(2) that is  
 187 valid under the assumption that the number of POs  
 188  $n$  tends to infinity and that the natural frequencies of  
 189 POs are randomly and uniformly distributed in the  
 190 intervals  $(a_j, b_j)$  ( $j = 1, \dots, k$ ). For simplicity of  
 191 formulas below we consider the case of one interval  
 192  $(a, b)$ . We also present simulation results in support  
 193 of theoretical approximations.

194 **3.1. Full synchronisation**

195 Consider a regime when all oscillators of system  
 196 (1)–(2) have the same constant value of the current  
 197 frequency, that is

198 
$$\omega = \frac{d\theta_i}{dt}, \quad i = 0, \dots, n. \quad (3)$$

199 We call this regime the *full synchronisation* (here  
 200 and in what follows synchronisation does not neces-  
 201 sarily imply in-phase dynamics).

202 Under condition (3) Eqs. (1) and (2) can be written  
 203 as

204 
$$\omega = \omega_0 + \frac{A}{n} \sum_{i=1}^n \sin(-\phi_i + \gamma), \quad (4)$$

205 
$$\omega = \omega_i + B \sin(\phi_i), \quad (5)$$

206 where  $\phi_i = \theta_0 - \theta_i$ . From (4)–(5) we can derive the  
 207 equation for the synchronisation frequency  $\omega$  by elim-  
 208 inating the variables  $\phi_i$ . In the limit  $n \rightarrow \infty$ , this  
 209 equation looks like

211 
$$\frac{\omega - \omega_0}{A} = -\frac{\cos \gamma}{B}(\omega - \bar{\omega})$$
  
 212 
$$- \frac{B \sin \gamma}{2(b-a)} \left[ f\left(\frac{\omega-b}{B}\right) - f\left(\frac{\omega-a}{B}\right) \right],$$
  
 213 
$$(6)$$

214 where

215 
$$\bar{\omega} = \frac{a+b}{2}, \quad f(x) = \arcsin x + x\sqrt{1-x^2}.$$

216 This is an implicit equation for  $\omega$  which depends on  
 217 several parameters. If all parameter values are fixed,  
 218 we can solve this equation and find the frequency of  
 219 full synchronisation (if there is a solution for these  
 220 parameter values).

221 **3.2. Partial synchronisation**

222 We say that a PO is partially synchronous with the  
 223 CO if there is a constant  $C$  such that for any time  $t$  the  
 224 difference between the phase  $\theta_0(t)$  of the CO and the  
 225 phase  $\theta_i(t)$  of the PO satisfy the inequality

226 
$$|\theta_0(t) - \theta_i(t)| < C. \quad (7)$$

227 We say that a system is in the regime of *partial*  
 228 *synchronisation* if some (but not all) of its oscillators  
 229 are partially synchronous with the CO. An important  
 230 fact obtained by computer simulations is that for large  
 231 values of  $n$  the current frequency of the CO in the  
 232 regime of partial synchronisation varies in a narrow  
 233 range around some constant value  $\omega$  due to averaging  
 234 of the influences of the oscillators that are not partially  
 235 synchronous with the CO. Basing on this observation,  
 236 we approximate the current frequency of the CO dur-  
 237 ing partial synchronisation by a constant  $\omega$  thus sub-  
 238 stituting  $\theta_0 = \omega t$  in (1)–(2). This allows us to derive  
 239 an equation for the frequency of partial synchronisa-  
 240 tion  $\omega$ .

241 Let us consider the case when  $a < \omega - B < \omega + B <$   
 242  $b$  (other cases of location of  $\omega$  relative to the ends  
 243 of the interval  $(a, b)$  can be considered in a similar  
 244 way). Then the equation for the frequency of partial  
 245 synchronisation  $\omega$  is given by the following formula  
 246 (see Appendix A for details):

247 
$$\frac{\omega - \omega_0}{A} = -\frac{\cos \gamma}{B}(\omega - \bar{\omega}) + \frac{B\pi \sin \gamma}{2(b-a)}$$
  
 248 
$$+ \frac{B \cos \gamma}{2(b-a)} \left[ g\left(\frac{\omega-a}{B}\right) - g\left(\frac{b-\omega}{B}\right) \right],$$
  
 249 
$$(7)$$
  
 250

251 where

252 
$$g(x) = x\sqrt{x^2 - 1} - \ln(x + \sqrt{x^2 - 1}). \quad (8)$$

253 Note that the set of POs that are partially synchronous  
 254 with the CO is formed by those POs whose natural  
 255 frequencies are distributed in the interval  $(\omega - B, \omega +$   
 256  $B)$ . Other POs will not be synchronous with the CO.

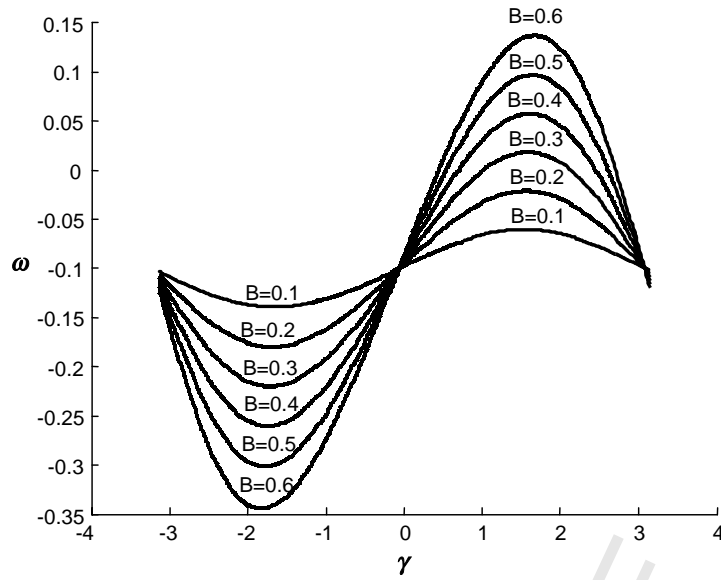


Fig. 2. Graphs of the frequency of partial synchronization  $\omega$  as a function of the phase shift  $\gamma$  for different values of the coupling strength  $B$ . The graphs are obtained by solving Eq. (7). Parameters of the system:  $\omega_0 = -0.1$ ,  $A = 0.5$ ,  $a = -1$ ,  $b = 1$ ,  $\bar{\omega} = 0$ .

257 Like Eq. (6), Eq. (7) is an implicit equation for  $\omega$   
 258 and its solution exists under some relations between  
 259 parameter values. If we fix an admissible set of pa-  
 260 rameter values, we can find the frequency of partial  
 261 synchronisation determined by Eq. (7).

262 Fig. 2 shows an example of the computation of  
 263 the frequency of partial synchronisation as a function  
 264 of the phase shift obtained by numerically solving  
 265 Eq. (7). The parameter  $A$  is fixed and the graphs in  
 266 Fig. 2 correspond to different values of the parameter  
 267  $B$ . It can be seen that the variation of the phase shift  $\gamma$   
 268 may lead to the significant deviation of the frequency  
 269 of partial synchronisation from the natural frequency  
 270 of the CO, and as a result to the change of the set of  
 271 POs included in the attention focus. For the same value  
 272 of  $\gamma$ , the deviation increases under greater values of  $B$ .

### 273 3.3. Attention focus formation and control

274 Suppose that the natural frequency of the CO is not  
 275 constant but changes according to the equation

$$276 \frac{d\omega_0}{dt} = -\alpha \left( \omega_0 - \frac{d\theta_0}{dt} \right), \quad (8)$$

277 where  $\alpha$  is a parameter that controls the speed of the  
 278 adaptation of  $\omega_0$  to the current frequency of the CO

$d\theta_0/dt$ . Such adaptation of the natural frequency of  
 279 phase oscillators has been used to increase the speed  
 280 of synchronisation and to get better tuning between  
 281 synchronous oscillators (Nishii, 1998; Hoppensteadt,  
 282 1992; Borisyuk et al., 2001). It is interesting to investiga-  
 283 te how (8) will influence the partial synchronisation  
 284 in the system (1)–(2).  
 285

286 Since the equilibrium of (8) in the regime of partial  
 287 synchronisation implies that  $\omega_0 = \omega$ , Eq. (7) takes the  
 288 form

$$290 \frac{\cos \gamma}{B} (\omega - \bar{\omega}) - \frac{B\pi \sin \gamma}{2(b-a)} - \frac{B \cos \gamma}{2(b-a)} \left[ g \left( \frac{\omega - a}{B} \right) - g \left( \frac{b - \omega}{B} \right) \right] = 0. \quad 291$$

292 This equation can be solved to find the value of  $\omega$ .

293 The important fact is that in this case the control of  
 294 the frequency of partial synchronisation by the phase  
 295 shift  $\gamma$  is easier. Relatively small changes of  $\gamma$  result in  
 296 large enough variation of  $\omega$  (note that  $\omega = \bar{\omega}$  for  $\gamma =$   
 297 0). We illustrate this by the results of the simulation  
 298 of system (1)–(2) and (8) shown in Fig. 3.

299 We consider a system with  $n = 1000$  POs whose  
 300 natural frequencies satisfy a uniform random distri-  
 301 bution in the interval  $(-1, 1)$ . The initial phases of

302 POs are randomly distributed in the interval  $(-0.5,$   
 303  $0.5)$ . The initial value of the natural frequency of the  
 304 CO and its initial phase are zero. In fact, the initial  
 305 values of the variables are not significant for the

dynamics of the system. The values of other parameters 306  
 are fixed and shown in the figure legend. The values 307  
 of the phase shift  $\gamma$ , which is a control parameter 308  
 in these simulations, vary in the interval  $(-\pi, \pi)$  and

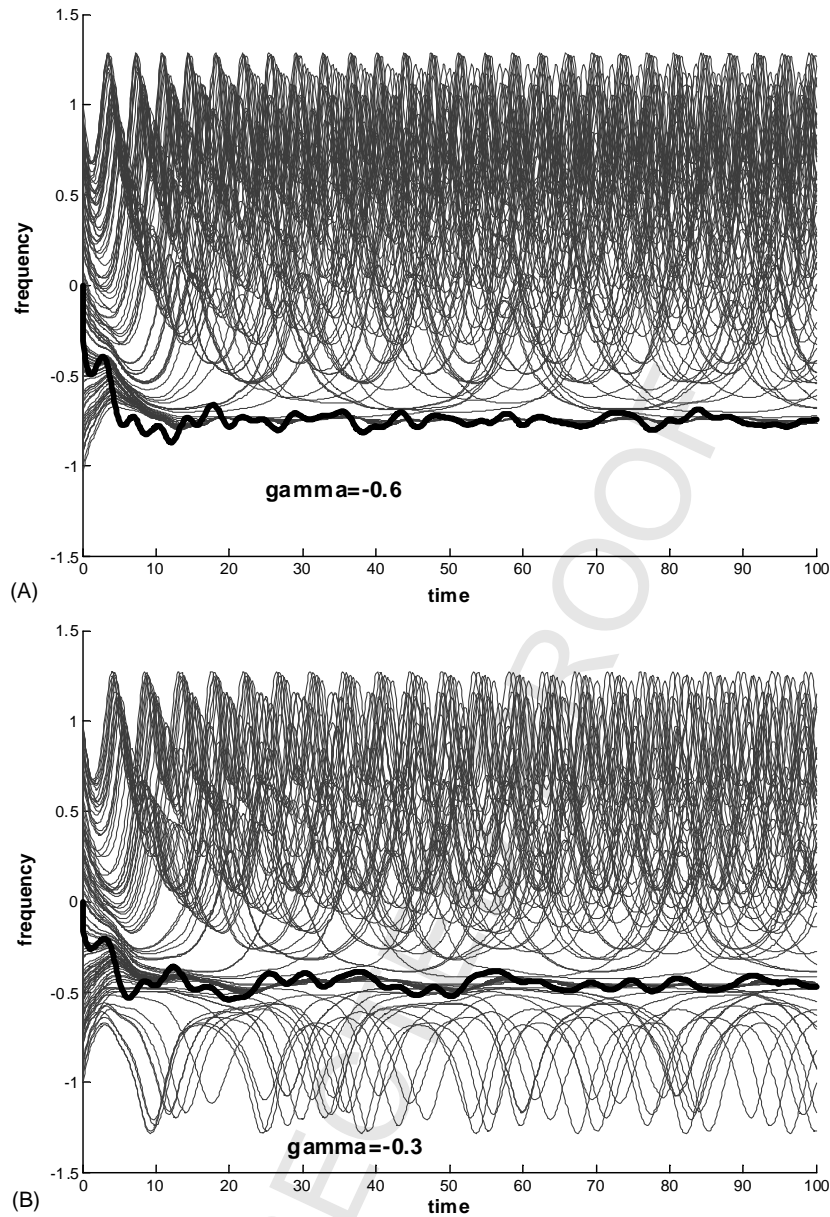


Fig. 3. Control of attention focus formation by phase shift variation. Each frame (A–D) shows the current frequencies of 100 POs randomly selected from 1000 POs participating in the simulations (thin lines) and the current frequency of the CO (thick line) vs. time. The phase shift variation allows attention switching from one group of POs to another. Examples of forming the attention focus by different sets of POs are shown for four selected values of the phase shift  $\gamma$  (A–D). Parameter values:  $A = 0.5$ ,  $B = 0.3$ ,  $a = -1$ ,  $b = 1$ ,  $\tilde{\omega} = 0$ .

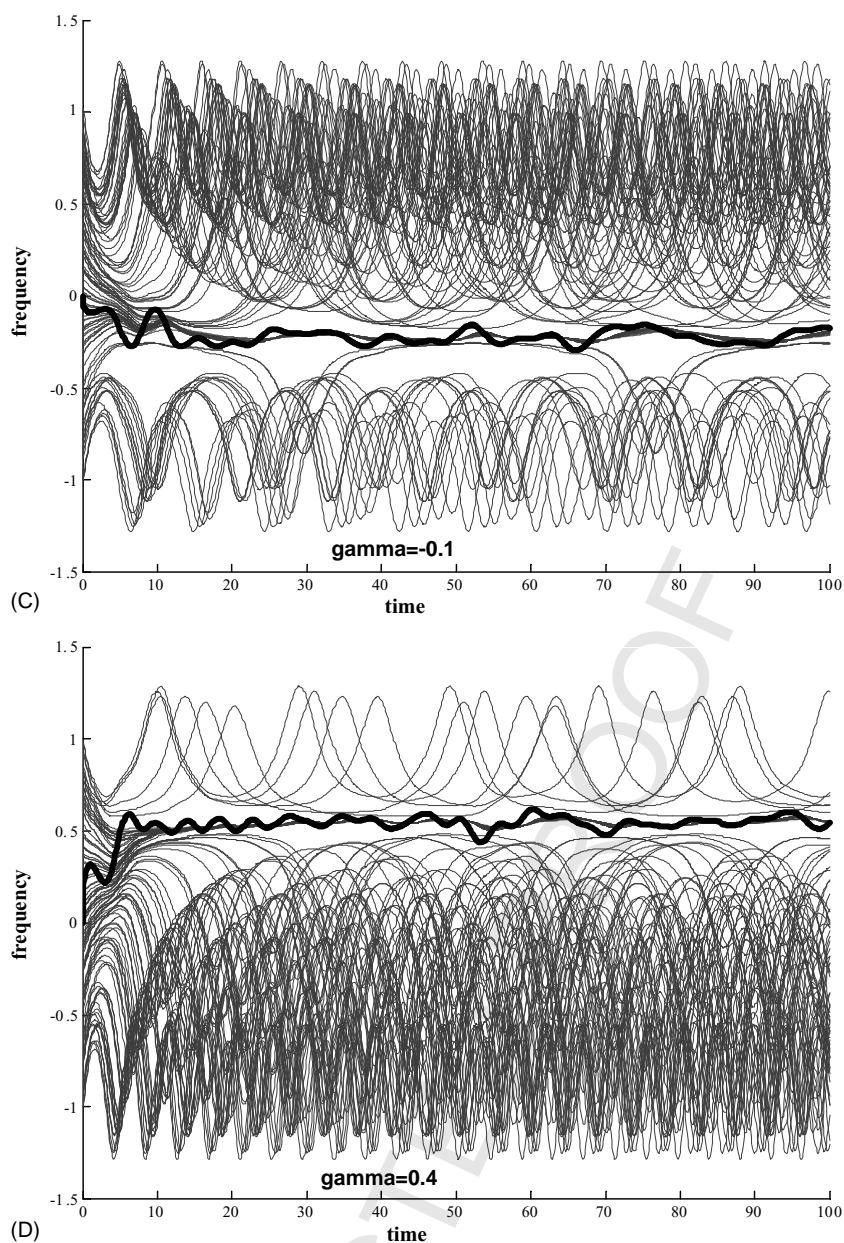


Fig. 3. (Continued).

309 control the choice of oscillators into the attention fo-  
 310 cus. Four examples of partial synchronisation and at-  
 311 tention focus formation with different groups of POs  
 312 are shown in Fig. 3A–D for four selected values of the  
 313 phase shift  $\gamma$ .

Fig. 3 shows the current frequency of the CO and 314  
 POs versus time. The trajectory for the CO is shown 315  
 by a thick line, the trajectories of POs are shown by 316  
 thin lines. It can be seen from the figure that the current 317  
 frequency of the CO is not constant but its varia- 318

tion caused by the influence of non-synchronous POs is relatively small. It would become even smaller for greater values of  $n$ . The CO engages into partial synchronisation a group of POs with the natural frequencies distributed in the interval of the length  $2B$  centred at  $\omega$ . Thus, the focus of attention represented by this group of POs depends on the length of the interval  $(\omega - B, \omega + B)$  and the density of the natural frequencies of POs in this interval. The current frequencies of POs that are not included in the attention focus are changing. The amplitude of the oscillations of their current frequency is equal to  $B$ , therefore a small number of the trajectories from time to time may come close to the trajectory of the CO (which means that the corresponding oscillators are synchronised by the CO) but then go away, others are always far away.

#### 4. Discussion

As we have found, the behavior of the ONN with the CO is properly reflected in the dynamics of the central element and in the case of distributed natural frequencies of oscillators this dynamics can be analytically described by averaging the influence on the central element from peripheral elements. Introduction of the phase shift results in significant variation of the frequency of synchronisation of the system. Thus, phase shifts can be used as controlling parameters in the formation of the attention focus in combination with the natural frequency of the CO or independently.

The regime of partial synchronisation represents a most general description of the intermediate state between full synchronisation and complex multifrequency or stochastic dynamics. The dynamics of the system in the regime of partial synchronisation can be very complex. Luckily, the important case of a large number of oscillators with randomly distributed natural frequencies is analytically tractable. The analysis shows that in this case the set of oscillators is divided into two subsets one of which shows nearly synchronous behavior. Introduction of the phase shift results in the changes of the synchronous subset and in the shift of the frequency of synchronisation.

In this paper we have considered the case when all one-directional connection strengths (backward or forward) are the same. This results in the formation of synchronous assemblies from POs whose natural

frequencies are distributed in some interval. In fact the same principles of analysis can be applied to the case when connection strengths are formed during previous learning or by the context. In this case an arbitrary set of partially synchronous oscillators can be formed.

The presented model proposes an efficient mechanism of feature binding in the attention focus which complies with the hypothesis that “spatially selective attention serves a useful role in collecting features from just one object at a time and delivering those features to later object recognition processes” (Wolfe and Cave, 1999). The binding of features is an indispensable component of several theories of attention, e.g. Treisman’s feature integration theory (Treisman and Gelade, 1980) or Wolfe’s guided search model (Wolfe, 1994; Chun and Wolfe, 2001). But this is only one aspect of attention modelling. Others include object recognition, memorisation and formulation of different strategies of object search. Thus, our model should be considered as an important component for these cognitive functions, but much more complicated model should be developed to cover paradoxical diversity of experimental evidence. The idea of synchronisation in an ONN with a central element turned to be useful in making first steps in this direction (Corchis and Deco, 2001; Kazanovich and Borisyuk, 2002)

#### Acknowledgements

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#### Appendix A

Let us divide the set of POs into two groups  $S$  and  $N$  for which the following inequalities take place

$$|\omega - \omega_i| \leq B, \quad i \in S, \quad (\text{A.1})$$

$$|\omega - \omega_i| > B, \quad i \in N. \quad (\text{A.2})$$

In case (A.1), a PO will work in the regime of partial synchronisation with the CO at the frequency  $\omega$ . In case (A.2), a PO will not be synchronous with the CO.

To obtain an equation for  $\omega$ , we should first find the mean values (over a period) of the functions  $h_i^1(t) =$

404  $\sin(\omega t - \theta_i(t))$ ,  $h_i^2(t) = \cos(\omega t - \theta_i(t))$  under con-  
 405 dition (A.2). It can be shown by using the technique  
 406 described in Kazanovich and Borisyuk (1999) that

$$408 \langle h_i^1(t) \rangle = \frac{1}{B}(\omega - \omega_i - \text{sgn}(\omega - \omega_i)\sqrt{(\omega - \omega_i)^2 - B^2}).$$

409 (A.3)

$$410 \langle h_i^2(t) \rangle = 0. \quad (A.4)$$

411 Averaging (1) by time gives the equation

$$412 \frac{\omega - \omega_0}{A} = R_S + \langle R_N \rangle, \quad (A.5)$$

413 where

$$415 R_S = \frac{1}{n} \sum_{i \in S} (-h_i^1(t) \cos \gamma + h_i^2(t) \sin \gamma),$$

$$416 R_N = \frac{1}{n} \sum_{i \in N} (-h_i^1(t) \cos \gamma + h_i^2(t) \sin \gamma).$$

417 Due to synchronisation of oscillators from  $S$  with  
 418 the CO, we can apply to the computation of  $R_S$  the  
 419 same approach that has been used when deriving the  
 420 equation for the frequency of full synchronisation. The  
 421 only difference is that the set of all POs is now replaced  
 422 by group  $S$ . Taking this into account, we obtain

$$424 R_S = -\frac{\cos \gamma}{B(b-a)} \int_S (\omega - x) dx$$

$$425 + \frac{\sin \gamma}{(b-a)} \int_S \sqrt{1 - \left(\frac{\omega - x}{B}\right)^2} dx. \quad (A.6)$$

426 Here  $S$  denotes the interval where the natural frequen-  
 427 cies of oscillators from  $S$  are located. By definition,  
 428  $R_S = 0$  if  $S = \emptyset$ .

429 Due to (A.3)–(A.4) we can obtain for  $\langle R_N \rangle$  under  
 430  $n \rightarrow \infty$  the formula

$$432 \langle R_N \rangle = -\frac{\cos \gamma}{B(b-a)} \int_N (\omega - x) dx$$

$$433 + \frac{\cos \gamma}{(b-a)} \left( \int_{N_1} \sqrt{\left(\frac{\omega - x}{B}\right)^2 - 1} dx \right.$$

$$434 \left. - \int_{N_2} \sqrt{\left(\frac{\omega - x}{B}\right)^2 - 1} dx \right), \quad (A.7)$$

where  $N_1 = (a, b) \cap (-\infty, \omega - B)$ ,  $N_2 = (a, b) \cap$  435  
 $(\omega + B, \infty)$ ,  $N = N_1 \cup N_2$ . By definition,  $R_N = 0$  if 436  
 $N = \emptyset$ . 437

Substituting the values of  $R_S$  and  $\langle R_N \rangle$  from (A.6) 438  
 and (A.7) into (A.5), we get 439

$$441 \frac{\omega - \omega_0}{A} = -\frac{\cos \gamma}{B}(\omega - \bar{\omega})$$

$$442 + \frac{\sin \gamma}{(b-a)} \int_S \sqrt{1 - \left(\frac{\omega - x}{B}\right)^2} dx$$

$$443 + \frac{\cos \gamma}{(b-a)} \left( \int_{N_1} \sqrt{\left(\frac{\omega - x}{B}\right)^2 - 1} dx \right.$$

$$444 \left. - \int_{N_2} \sqrt{\left(\frac{\omega - x}{B}\right)^2 - 1} dx \right) \quad (A.8)$$

The values of the integrals in (A.8) can be computed 445  
 depending on the relation between the interval  $(a, b)$  446  
 (where the natural frequencies of the POs are located) 447  
 and the interval  $(\omega - B, \omega + B)$  (where, according to 448  
 (A.1), the natural frequencies of the POs from  $S$  are 449  
 located). For example, for the case  $a < \omega - B <$  450  
 $\omega + B < b$  (A.8) takes the form (7). 451

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